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GROWTH OF WELL CHILDREN: ANALYSIS OF STATURE AND WEIGHT, 3 MONTHS TO 13 YEARS

By

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(*Received February 23, 1938*)

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INTRODUCTION

In 1929 the Brush Foundation and the Spelman Fund jointly made possible at Western Reserve University a long term study of developmental growth in children which should form the nucleus of an investigation into the unfolding of personality. This was designed to be enlarged as funds became available for complementary studies upon aspects other than those of physical growth, physical maturation and mental expansion. The unequalled resources of the Anatomical Laboratory provided a background of information from the bodies of dead children whose stage of progress attained during life could be checked against that of the living at the same age in order to ascertain the effect of acute

disease and chronic constitutional handicaps upon the patterns of growth. Other organizations made special fields of investigation their particular responsibility so that the conjoint endeavor, known as the Developmental Health Inquiry of the Associated Foundations, now covers the life span between conception and early married life.

As the work progressed, special additional funds were provided by the General Education Board and the Cleveland Foundation. To these also we therefore acknowledge our indebtedness for facilities and support in carrying on the particular work here recorded. By good fortune the Inquiry became as much a feature of social life in the Community as the Junior League. It commanded the interest of the economically stable, more intelligent and cooperative families of Cleveland and its suburbs and has been fostered by every Board of Education and Private School. In the course of time we shall be able to present complete registers of childhood in a sufficient number of families where health, training and social acceptance ensure regularity of record to justify confidence in deduction on the progress of developmental growth in each of its diverse aspects.

Other groups of observations are larger, such as that great series patiently accumulated by Dearborn (7). This of ours does not pretend to vie with such but, being drawn only from a select and highly cooperative class, does provide a sample which illustrates developmental growth under intelligent parental guidance and efficient medical supervision. It provides also an opportunity for correlation between studies in different aspects of development greater than any investigation hitherto undertaken.

To await the completion of this comprehensive survey before issuing reports on special features of the study involves risks greater than those tempted by the interim publication of observations to date. In this article we present the information obtained on height and weight from serial examination of children from birth to seven years amplified by records from serial examinations of older siblings and other children up to the age of thirteen years.

TECHNIQUE OF MEASUREMENT AND APPRAISAL OF HEIGHT AND WEIGHT AS GROWTH INDICATORS

Regular examinations are given at three, six, nine and twelve months, then every six months until and including the fifth birthday and after that, at yearly intervals within a week of the birthday.

Height is to be differentiated from horizontal length. It is measured by an anthropometer of the Martin design (14). Horizontal length, not used in this article except during the first year of life, is obtained by measurement of the child recumbent on the measuring couch (see Thompson, 17). The child whose shoes have been removed stands on the floor, as erect as he can, not against a wall. The foot of the anthropometer is rested on the floor behind him or by his side, the lower transverse arm is adjusted so that it just clears the vertex of his head and the measurement is read in millimeters directly on the stem of the instrument. The ascertained height is called to a recorder who writes it down immediately. The records of the last previous examination lie before the re-

corder when the measurement is registered, any incongruity being reported to the measurer at once. Many other bodily measurements are recorded on the anthropometric sheet at the same examination. The sheet is given to the cartographer who constructs therefrom the mannikin (Fig. 1) on millimeter paper. The scale used is 1 mm. to 1 cm. The mannikins representing successive examinations show at a glance possible errors of measurement or record and changes in posture of the child. Weight, technique of recording which is identical with that of height, is registered on a Fairbanks scale after removal of shoes but not of clothes. The clad weight of a child differs little between summer and winter unless specially heavy garments (e.g., corduroy trousers) are worn. Even

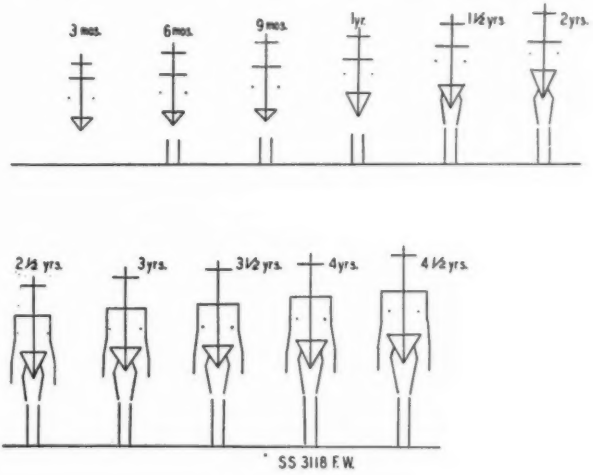

FIG. 1. Mannikin.

then the difference in weight is little more than may be expected from divergence in time of day or physiological condition. Such seasonal differences as may appear in the weight of any child are due not to changes in clothing but to fluctuations in hydration of bodily tissues. These are recorded as changes in relative density of subcutaneous tissues visible in the soft tissue roentgenograms made of the child as part of the examination.

Differences in stature or standing height are checked against differences in horizontal length. Thus changes in posture are recorded. If increase in stature is greater than that in horizontal length the child is holding himself better than before. If however increase in stature is less than that in horizontal length the child's posture is poorer for he does not hold himself so well. Thus health and emotional condition affect stature. The only exception to this generalization is found in children whose increased size of buttocks or eagerness to cooperate on the horizontal length measurement results in a position of temporary opisthotonos which somewhat reduces the horizontal length.

In the analysis of stature differentiation must be made between increments of leg length and of trunk length. The relation between these two differs in successive periods of childhood. The later increments of stature in adolescence, for example, are wholly confined to trunk. In earlier childhood divergencies from the usual relationship between increments in trunk and in leg length are determined by factors which, for want of a better designation, are today recorded as of modulations in endocrine balance. In this paper we do not consider differentiation of stature increment into its components.

Weight is not a true measure of growth as Hammett has recently pointed out (12, 13). It is a record of body bulk and has three components, namely, increase of supportive tissues, mainly muscle and bone, in addition to organ growth; accumulation of fat; and modifications in hydration. While steady increase is due to the first and second components, diurnal fluctuations, and a large part of those fluctuations recorded as seasonal, are due to differences in hydration. While increments of stature, with those reservations noted above, register true body growth, alterations in weight are more nearly a register of health.

During the past seven years many groups of children have undergone examination of the type here recorded but are not included in this particular series. Our observations on the additional groups have been used, like those made on our dead children, as checks on the standard series here presented. Some of these are utilized later in this article.

REDUCTION OF DATA

In reducing our raw data, the punched card method has been used throughout. For this purpose International Business Machine (Hollerith) equipment has been made available to the Laboratory through the kindness of Mr. T. J. Watson and Mr. R. G. Thomas, Jr. of the International Business Machines Corporation.

In order to save space on our 80-column cards and decrease the time consumed in mechanical operations, measurements are coded to figures within the range 00 to 99 before cards are punched. The size of the class interval necessarily varies from measure to measure. For the measures discussed here, the coding is as follows: weight, from three months to four and a half years inclusive, is classified in one-half pound intervals; from five years to ten years inclusive, in one pound intervals; from eleven years upward in two pound intervals. For stature and horizontal length the same codes are used: from three months to nine years inclusive, the class interval is 5 mm.; from ten years upward the class interval is ten mm. All codes are noted alongside the raw measurements on our permanent statistical records. Each of these records has space for nine serial examinations; each examination comprises approximately 100 measures or descriptive classifications and each measure is provided with columns for raw data, converted data and coded data. From these records it is easy to transfer the coded data to punched cards in whatever number and combination of measures desired. We now have in use more than 30 master code cards, each one devised to carry a group of measurements for a particular purpose. When the cards are

punched and verified the summations needed for our formulae are obtained from the International Business Machines tabulator sheets and final computations are made with Monroe calculators. All operations are carried out in duplicate for checking. Space does not permit publication of our raw data or our coded data, all of which are a part of our permanent records. All statistical constants have been derived by use of standard formulae which are explained in the statistical references cited.

SAMPLING AND COMPARISON OF OUR STANDARD INQUIRY SERIES WITH THOSE OF OTHER INVESTIGATORS

The homogeneity of our population is primarily economic and social. The children admitted to our Inquiry standard series are, with few exceptions, ultimately of North European ancestry. Homogeneity of stock, however, has been accidental and incidental; the important factors operating to integrate our group are first, the superior or at least secure economic position of the parents and secondly, voluntary participation by the parents in a long term program of remeasurement. Eighty percent of our children's parents are engaged in professional and business executive occupations; the other twenty percent are in commercial and managerial occupations. There are no laborers. The one factor in the selection of our sampling which is held constant is that of willingness on the part of the parents to avail themselves of this community resource for the physical and behavioral appraisement of their children, an appraisement which is communicated to them by their family physician or pediatrician. We believe this homogeneity of financial, educational and cultural advantages is a more valid basis for sampling than is racial homogeneity (when racial homogeneity can be found in the United States) from the point of view of studying group behavior. This paper, however, is no contribution to the discussion of heredity versus environment; our population may be as homogeneous in hereditary characters as it is in environmental characters or the two may be too interrelated for identification. Our selection is not made on the basis of either type of character but on the basis of membership in a group as it functions in society; this membership is undoubtedly determined by both types of characters.

Our "modified longitudinal" method of selection of children for age standards has been mentioned. The ideal of an entirely longitudinal study from five years of age to the completion of growth cannot be attained in this laboratory until 1944. Of the children whose measurements are used in the reduction of our present standards, the number entering at three months is far greater than the number entering at any other age. Of 459 girls used in the standards, 116 or 25 percent entered at three months; the remaining 343 girls entered at one or other of the succeeding regular examination ages. Tables I and II show the number of children who have undergone examination over a specified period of time. The figure at the interception of a horizontal row and a vertical column denotes the number of children who have had regular serial examinations without interruption from the age on the row to that on the intersecting column.

From Table I we find that while 130 boys have had examinations at three

months and six months, there are, so far, 37 boys who have had examinations at every regular, set serial date from three months to five years. In the column at the extreme left is recorded the number of examinations given at each successive age level; these figures do include some children who have had one examination only, but the number is so small that it has no influence upon the statistical results or interpretations of this paper. At no age is there a complete change in population; at no age is the number entering as great as the number re-examined from the previous period (with the exception of three months).

TABLE I. RECORD OF UNINTERRUPTED SERIAL EXAMINATIONS (BOYS). THE FIGURE AT THE INTERCEPTION OF A HORIZONTAL ROW AND A VERTICAL COLUMN DENOTES THE NUMBER OF CHILDREN WHO HAVE HAD REGULAR SERIAL EXAMINATIONS WITHOUT INTERRUPTION FROM THE AGE ON THE ROW TO THAT ON THE INTERSECTING COLUMN

	Age	6	9	12	18	2	2½	3	3½	4	4½	5	6	7	8	9	10	11	12	13	14	15	16	17	18
137	3	130	121	115	104	99	91	80	65	57	49	37													
147	6	...	137	131	120	113	105	94	79	71	63	51	1												
151	9	...	...	141	128	120	112	101	86	78	70	58	3												
154	12	...	...	...	141	131	122	110	95	87	78	66	10												
152	18	...	...	...	...	142	133	120	104	96	86	74	18	1											
157	2	...	...	...	...	...	146	134	116	108	98	86	30	11											
164	2½	...	...	...	...	...	...	145	125	116	106	94	38	17											
168	3	...	...	...	...	...	...	...	141	131	118	103	46	25	7										
164	3½	...	...	...	...	...	...	...	...	146	129	111	54	32	11										
157	4	...	...	...	...	...	...	...	...	...	141	120	60	38	17	3									
184	4½	...	...	...	...	...	...	...	...	...	...	141	77	55	33	17	3								
160	5	...	...	...	...	...	...	...	...	...	...	...	95	72	49	32	14	1							
113	6	...	...	...	...	...	...	...	...	...	...	...	..	87	64	46	25	5							
104	7	...	...	...	...	...	...	...	...	...	...	...	..	..	75	56	34	15	3						
88	8	...	...	...	...	...	...	...	...	...	...	...	..	..	..	65	43	24	11	3	2				
77	9	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	50	30	17	9	5	1			
100	10	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	68	35	18	12	6	1		
99	11	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	62	27	14	8	2	1	1
106	12	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	64	31	9	2	1	1
71	13	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	..	37	12	5	3	1
	14	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	..	..	16	7	5	1

However, due partly to the fact that first examinations of those entering at any one age may have taken place at any time during the past six calendar years, our population does change to the extent that among the children above ten years of age there is none who was examined below five years of age. The figures in these tables are correct to July, 1937.

Over the period three months to three and one half years of age 1386 examinations have been made on 257 girls, 1394 examinations on 266 boys. From four to nine years of age, 241 girls have had 925 examinations, 250 boys have had 883 examinations. From ten to thirteen years of age, 165 girls have had 325 examinations, 195 boys have had 376 examinations.

Of the 241 girls examined between four and nine years of age, 125 or 51.9 percent commenced their examinations at some time during the earlier age period, i.e., three months to three and one half years; 72 or 29.9 percent are sisters of girls in the earlier age period while 44 or 18.2 percent were neither themselves examined nor had sisters in the earlier age period. Of the 165 girls examined between ten and thirteen years of age, 38 or 23 percent were also

examined in the four to nine year period; 29 or 17.6 percent are sisters of the girls in the four to nine year period and 98 or 59.4 percent have neither examination nor sister in the previous age period. Fifteen of the older girls are sisters of girls entered during the first period, namely three months to three and one half years.

Of the 250 boys examined between four and nine years of age, 97 or 38.8 percent were also examined during the earlier period, i.e., three months to three and one half years; 62 or 24.8 percent are brothers of the boys in the earlier

TABLE II. RECORD OF UNINTERRUPTED SERIAL EXAMINATIONS (GIRLS). THE FIGURE AT THE INTERCEPTION OF A HORIZONTAL ROW AND A VERTICAL COLUMN DENOTES THE NUMBER OF CHILDREN WHO HAVE HAD REGULAR SERIAL EXAMINATIONS WITHOUT INTERRUPTION FROM THE AGE ON THE ROW TO THAT ON THE INTERSECTING COLUMN

	Age	6	9	12	18	2	2½	3	3½	4	4½	5	6	7	8	9	10	11	12	13	14	15	16	17
116	3	113	110	104	99	92	83	78	65	61	48	36												
132	6	...	129	121	116	107	96	90	77	72	59	41	1											
141	9	...	...	130	123	113	102	96	83	78	65	47	5											
118	12	...	...	...	139	128	114	106	93	88	75	54	8	1										
149	18	...	...	...	...	141	126	118	105	100	85	61	14	1										
165	2	...	...	...	...	...	150	140	127	121	104	79	32	17	1									
170	2½	...	...	...	...	...	...	156	140	133	115	90	42	27	2									
183	3	...	...	...	...	...	...	...	159	150	130	103	53	38	10									
182	3½	...	...	...	...	...	...	...	...	166	144	116	64	49	21	2								
179	4	...	...	...	...	...	...	...	...	...	166	136	82	66	36	17								
200	4½	...	...	...	...	...	...	...	...	...	...	152	96	80	48	27	2							
172	5	...	...	...	...	...	...	...	...	...	...	...	110	93	59	37	10	2						
121	6	...	...	...	...	...	...	...	...	...	...	...	...	102	64	42	15	5	1					
114	7	...	...	...	...	...	...	...	...	...	...	...	...	...	71	48	20	10	6					
73	8	...	...	...	...	...	...	...	...	...	...	...	...	...	..	57	29	18	12	1				
66	9	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	37	26	19	7	3	1		
88	10	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	61	27	9	4	1		
101	11	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	59	20	6	3	1	
88	12	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	44	14	5	3	1
48	13	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	19	8	5	2

age period while 91 or 36.4 percent were neither themselves examined nor had brothers in the earlier age period. Of the 195 boys examined between ten and thirteen years of age, 50 or 25.6 percent were also examined during the four to nine year period; 35 or 17.9 percent are brothers of the boys in the four to nine year period while 110 or 56.4 percent were neither themselves examined nor had brothers in the previous age period. Twenty of the older boys are brothers of boys entered during the first period, namely three months to three and one half years inclusive.

We have used, then, 2636 examinations made upon 459 girls who represent 373 families; 2653 examinations made upon 493 boys who represent 409 families. Taking into consideration the boy-and-girl siblings, we may say that we have used 5289 examinations made upon 952 children who represent 647 families.

We are aware that this "modified longitudinal" procedure may be criticized statistically with justification on the ground that it produces neither a series nor a cross section; that serial constants may be modified by the addition of new children, whereas cross section constants may be distorted by undue weighting

at some points on the distribution caused by the reappearance at successive examinations of the same children. Nevertheless, while we are engaged in assembling a truly longitudinal study we must have some working sample for appraisal purposes and the sampling here presented appears to be sufficiently homogeneous throughout its age range to balance any effects of modification of its longitudinal character.

In Figure 2 A (male) and B (female), are given the curves of mean stature of 37 boys and 36 girls whose records are complete at every examination from

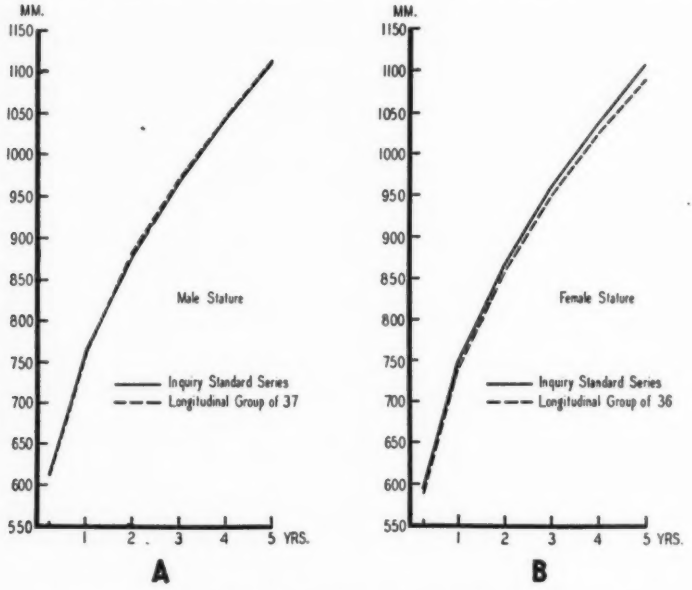

FIG. 2. Comparison of Inquiry standards of stature (solid line) with mean growth curves (broken line) of special groups examined at all twelve regular examination dates from three months to five years inclusive. A. Thirty seven boys, B. Thirty six girls.

three months to five years for comparison with our stature standards over the same period of time. There is no significant difference between the curves either of the boys or of the girls.

We are reproducing, for comparison, in Figures 3, 4, 5 and 6 the stature and weight curves of some other investigators, namely Gray and Ayres, Baldwin, Woodbury, Bayley and Davis, Meredith, Boynton, Boas, and Richey, together with our own.

We regret that it has been impossible to use in this study certain important data for comparison carefully gathered by May Ayres Burgess, by Baldwin, by Shuttleworth and by the Merrill Palmer School which do not happen to be presented in form suitable for direct comparison with those here included.

In comparing these curves the most noticeable differences are the quantitative differences between the mean values; in addition, however, there are other differences striking enough to prohibit the expression of these curves by any common formula. The curves differ markedly in the degree of convexity (stature) and concavity (weight) and in magnitude and site of "spurt" and "slump." There is a difference in the age range over which the curves are approximately

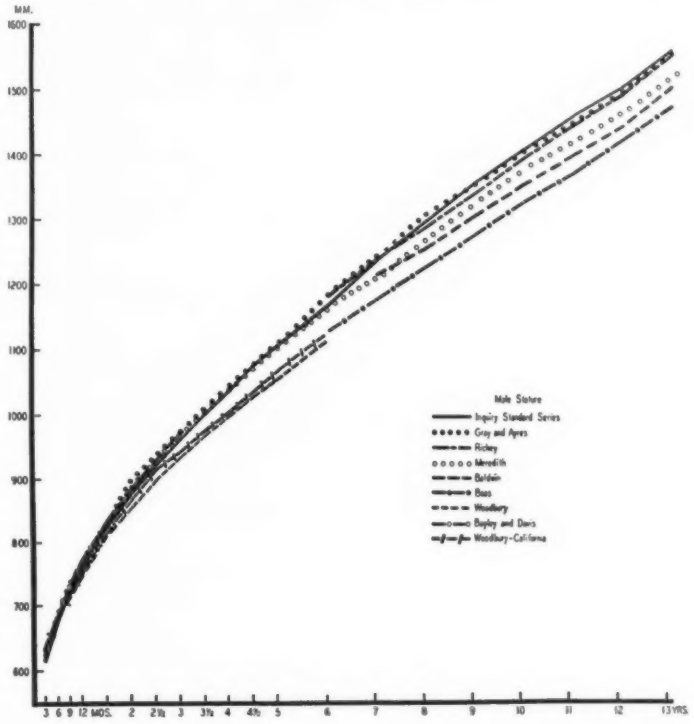

FIG. 3. Comparison of Inquiry standards of stature (solid line), boys, with curves constructed from figures presented by others.

logarithmic. There is a difference in the degree of increment and in the age of its occurrence during adolescence. It is evident, then, that these investigators used for their studies widely differing samplings which produced as many patterns of average stature and weight as there were samples used. Since the value of any statistical study is fundamentally affected by the sampling method it is the sampling which should be first evaluated in appraising the study.

The displacement in space of curves representing different samplings is a normal expectation since we know that there is a wide range in stature and weight and that no two samples would be likely to be drawn from exactly the same

section of the total distribution. The meaning to be assigned to other differences, however, is doubtful. For instance, it is impossible to say whether a deflection in the curve represents a true slump in annual increment or whether the investigator may have permitted a significant population change to enter into his sampling between one age and the next. It seems possible that this question of selection has not always been given the consideration it deserves. In a problem

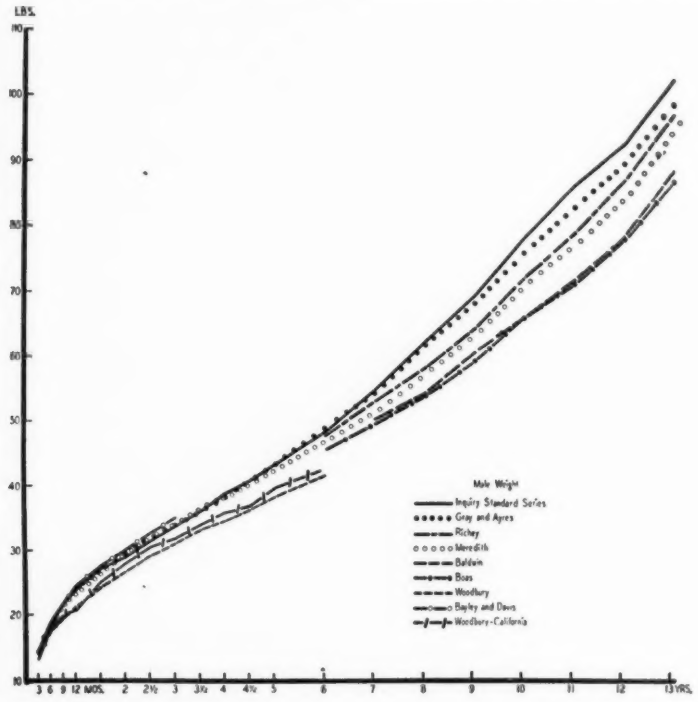

FIG. 4. Comparison of Inquiry standards of weight (solid line), boys, with curves constructed from figures presented by others.

of this nature, the large, cross-section type of sampling is of doubtful value because the entire population which this represents cannot function as a unit in the study of growth. For practical purposes the growth pattern as represented by the usual random sample is likely to be biased or distorted by a preponderance of individuals from one or the other extreme of the distribution as the curve progresses from age to age.

Further understanding of the differences between our series and these other series may be obtained from the critical ratios in Table III. Accepting as significant a critical ratio of three or more between the difference of the means and the standard error of this difference, the relationship between the several series

is particularly interesting. At three months of age, the mean horizontal length of the Inquiry girls is lower than those of Woodbury, Boynton, or Bayley and Davis, but the difference is significant only in comparison with the mean length cited by Woodbury. At six months and at nine months the Inquiry-Woodbury critical ratio drops below the criterion of significance, but by twelve months the critical ratio is again significant. By this age, however, the Inquiry horizontal

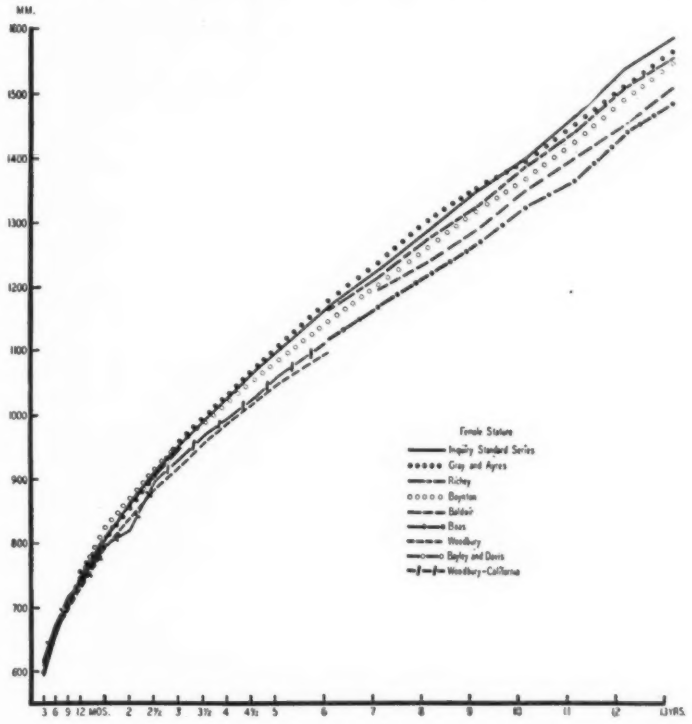

FIG. 5. Comparison of Inquiry standards of stature (solid line), girls, with curves constructed from figures presented by others.

length is higher than the Woodbury horizontal length, a reversal of the earlier relationship. At three years of age the differences between the Inquiry series and those of Bayley and Davis, and of Boynton are still insignificant. The Inquiry-Woodbury critical ratio, however, has risen to 12.39. At five years the difference in female stature between the Inquiry series and that of Gray and Ayres is insignificant, whereas the mean of the Inquiry series is now significantly higher than that of the Boynton series, showing a critical ratio of 3.38; the Inquiry-Woodbury difference shows a critical ratio of 14.12. At eleven years the Inquiry-Boynton critical ratio has risen to 5.05, the Inquiry-Gray and Ayres means are

still closely related and the Inquiry-Boas female stature means show a difference whose critical ratio is 13.51. Between the Inquiry series and the Gray and Ayres series, in both of which economic superiority is largely the criterion of selection, the only significant difference occurs at age twleve, when the critical ratio is 3.36. We anticipate that this difference will lose its significance when our adolescent series has obtained its full numerical strength.

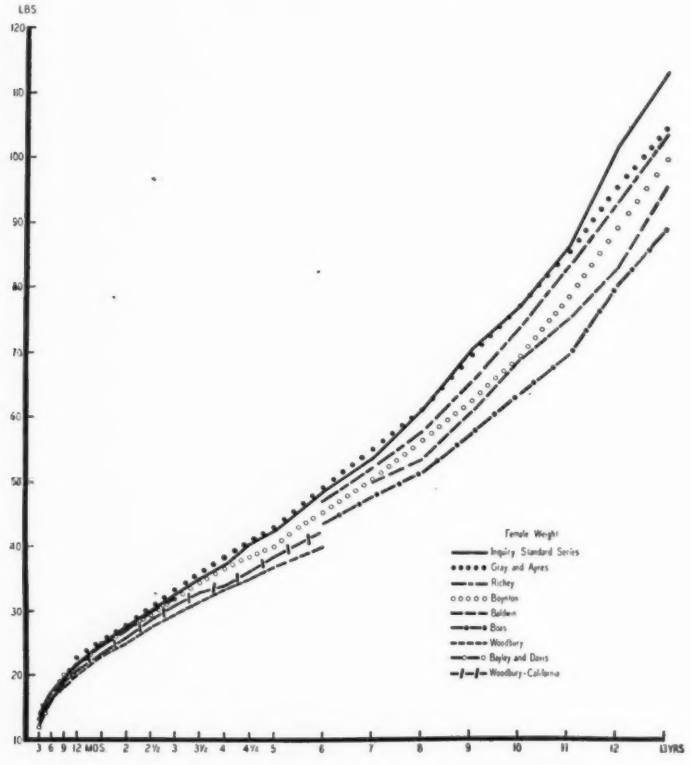

FIG. 6. Comparison of Inquiry standards of weight (solid line), girls, with curves constructed from figures presented by others.

The crossing of the curves of means, namely a situation wherein the mean of one series from first being significantly greater than the mean of a second series, becomes so closely related to the second series that the difference is insignificant and still later becomes significantly lower than that of the second series, can be explained by one of two possibilities:

1) both series of means may have been computed from samplings homogeneous throughout their age ranges. The two curves must then represent a real difference between the growth patterns of two distinct types of population.

2) the sampling in one series or the other or in both may have been permitted to lose its homogeneity from age to age. The curve of means which loses its homogeneity fails to represent any true growth pattern and becomes an artifact.

We are presenting in Figure 7 the Inquiry standards of male stature with stature means of several other groups examined by us. One group is composed

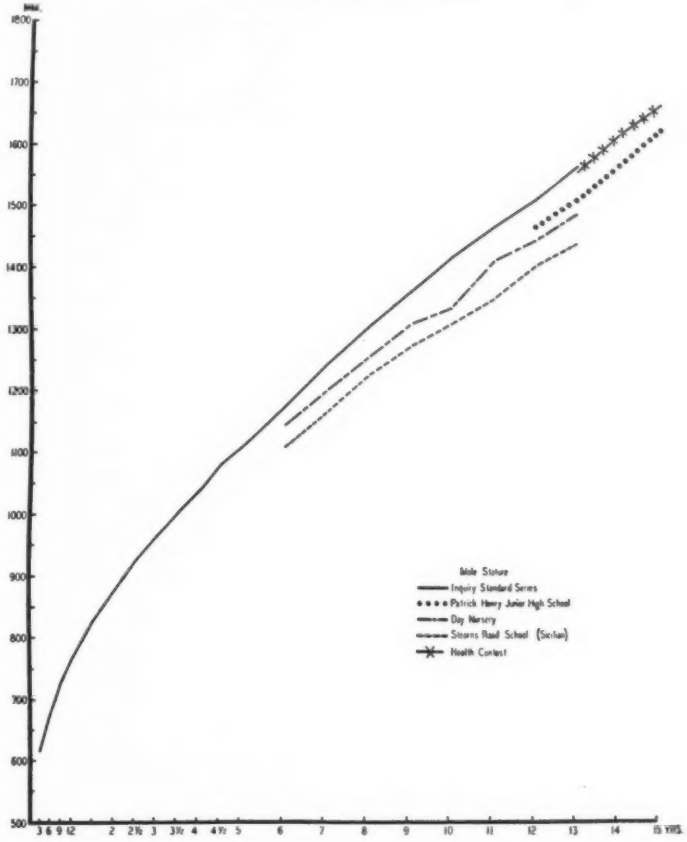

FIG. 7. Comparison of Inquiry standards of male stature with means of other groups examined by us.

of boys of Sicilian parentage, a second group is from the Day Nurseries of Cleveland, a third group is from one of the large junior high schools of Cleveland, a fourth group comprises the boys voted by the Cleveland Public School children as the most representative for entry in a Health Contest. The distinction of these groups is immediately apparent. The position each occupies on the stature distribution is logical. The Sicilian group, its homogeneous population present-

TABLE III. SIGNIFICANCE OF DIFFERENCES IN MEAN STATURES OF THE INQUIRY SERIES AND SERIES PRESENTED BY OTHER INVESTIGATORS

Age	Inquiry—Gray & Ayres CR	Inquiry—Bayley & Davis CR	Inquiry—Woodbury CR	Inquiry—Boynton CR	Inquiry—Boas CR
3 Months		0.95	7.35	1.14	
6 "		1.80	2.85	1.01	
9 "		0.17	2.23	0.12	
1 Year	0.88	0.44	5.33	0.40	
2 "	0.95	0.15	8.78	1.90	
3 "	0.47	0.62	12.39	0.24	
4 "	0.51		13.63	1.66	
5 "	1.06		14.12	3.38	
6 "	1.28		14.09	4.63	8.81
7 "	1.34			4.04	9.59
8 "	1.75			3.73	9.65
9 "	.30			4.17	11.57
10 "	.76			4.39	10.53
11 "	1.82			5.05	13.51
12 "	3.36			8.17	13.00
13 "	2.60			4.20	10.79

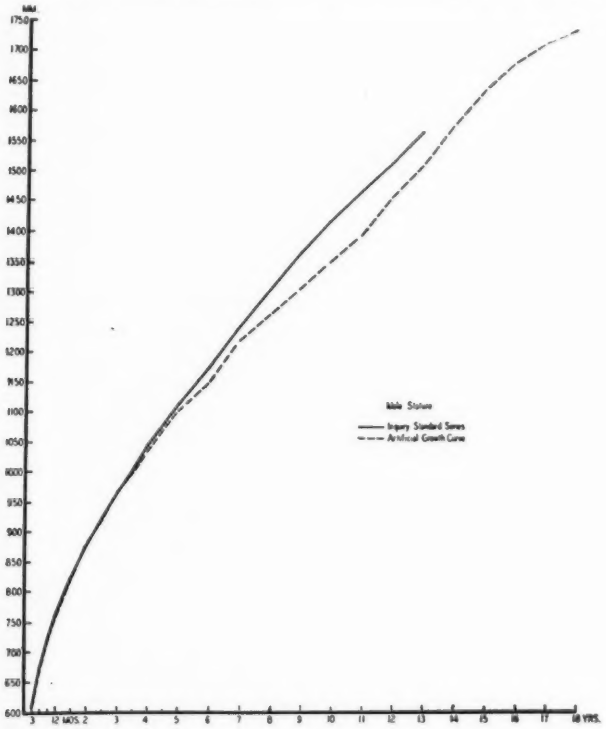

FIG. 8. Curve of mean stature derived from the combination of five different samplings of boys measured in the Inquiry investigation.

ing a smooth curve, is lowest; the day Nursery group is higher, but its heterogeneous composition causes it to present an irregular curve; the junior high school group is centrally located and its curve is smooth; the Health Contest group approaches the stature of our standard group and it is reasonable to believe size would be a factor influencing choice in a Health Contest.

As an example of the type of curve resulting when significantly different populations are combined, we have plotted in Figure 8 the curve of mean stature derived from combining all five groups. The inflections, concave or convex to the base line, presented by this curve as the total population comes to include a disproportionate number of short or of tall boys provide us with a well marked

TABLE IV. INQUIRY STANDARDS MALE STATURE. THE MEANS WITH THE NUMBER OF CHILDREN ON WHICH EACH IS BASED, THE STANDARD DEVIATIONS, COEFFICIENTS OF VARIABILITY AND THE RELIABILITIES OF THESE CONSTANTS

Age	N	Mean mm.	PE	Standard Deviation	PE	Coefficient Variability	PE
3 Months	137	613.46	.89	15.36	.63	2.50	.10
6 "	147	672.28	1.28	23.00	.90	3.42	.13
9 "	151	719.73	1.24	22.52	.87	3.13	.12
12 "	154	761.36	1.43	26.34	1.01	3.46	.13
18 "	152	822.25	1.65	30.09	1.16	3.66	.14
2 Years	157	874.33	1.73	32.17	1.22	3.68	.14
2½ "	164	922.07	1.69	32.14	1.20	3.49	.13
3 "	168	962.25	1.79	34.44	1.27	3.58	.13
3½ "	164	1001.76	1.89	35.91	1.34	3.58	.13
4 "	157	1039.65	1.99	36.93	1.41	3.55	.14
4½ "	166	1077.54	2.06	39.39	1.46	3.66	.14
5 "	160	1109.38	2.20	41.20	1.55	3.71	.14
6 "	113	1172.38	2.75	43.37	1.95	3.70	.17
7 "	104	1238.93	3.25	49.17	2.30	3.97	.19
8 "	88	1301.34	3.70	51.50	2.62	3.96	.20
9 "	77	1359.80	3.78	49.13	2.67	3.61	.20
10 "	100	1413.80	3.62	53.73	2.56	3.80	.18
11 "	99	1465.20	4.34	63.74	3.06	4.35	.21
12 "	106	1510.85	4.77	72.74	3.37	4.81	.22
13 "	71	1567.11	6.37	79.58	4.50	5.08	.29

example of a familiar "growth curve" which is really an artifact. These five groups were selected for study as separate units and were not intended to be combined. However, it is easy to understand how a similar if less severe sampling error is almost inevitable when the personnel of the age groups changes from age to age, unless the investigator carefully plans his selection so that his criterion is applied uniformly at all age levels. When dealing with very large groups, this control is difficult to achieve. The smaller sampling, homogeneous both at the age level and within the age range, is for practical purposes a prerequisite for the reduction of meaningful and comparable constants.

PRESENTATION OF CONSTANTS AND VALIDATION OF TIME INTERVALS BETWEEN EXAMINATIONS

The stature and weight constants computed from our data complete to July 1, 1937 appear in Tables IV, V, VI and VII. These constants are final and adequate for use as standards of appraisal from three months to five years of age;

TABLE V. INQUIRY STANDARDS MALE WEIGHT. THE MEANS WITH THE NUMBER OF CHILDREN ON WHICH EACH IS BASED, THE STANDARD DEVIATIONS, COEFFICIENTS OF VARIABILITY AND THE RELIABILITIES OF THESE CONSTANTS

Age	N	Mean lbs.	PE	Standard Deviation	PE	Coefficient Variability	PE
3 Months	137	14.37	.08	1.40	.06	9.75	.40
6 "	147	18.52	.10	1.83	.07	9.87	.39
9 "	151	21.64	.13	2.30	.09	10.63	.42
12 "	154	23.85	.13	2.45	.09	10.29	.40
18 "	154	26.09	.15	2.73	.11	10.11	.39
2 Years	157	29.05	.16	2.99	.11	10.30	.40
2½ "	164	31.28	.18	3.32	.12	10.63	.40
3 "	168	33.54	.17	3.33	.12	9.94	.37
3½ "	164	35.92	.20	3.85	.14	10.72	.40
4 "	158	38.39	.24	4.49	.17	11.69	.45
4½ "	165	40.72	.23	4.37	.16	10.74	.40
5 "	160	43.16	.27	5.13	.19	11.89	.45
6 "	113	48.53	.38	5.96	.27	12.28	.56
7 "	104	54.65	.45	6.77	.32	12.39	.59
8 "	88	62.24	.68	9.44	.48	15.17	.80
9 "	77	69.53	.73	9.52	.52	13.69	.76
10 "	100	78.54	.90	13.28	.63	16.91	.83
11 "	99	86.49	1.13	16.57	.79	19.16	.96
12 "	106	92.69	1.17	17.85	.83	19.26	.92
13 "	71	102.80	1.47	18.41	1.04	17.91	1.05

the constants from six years upwards are currently being used as standards but, as more measurements become available at these ages, we anticipate slight changes in values particularly in the age constants above ten years. For our own appraisement purposes we have extended our table of standards tentatively through the adolescent ages by using the Gray and Ayres constants because they approximate our own more nearly than any others in the literature.

TABLE VI. INQUIRY STANDARDS FEMALE STATURE. THE MEANS WITH THE NUMBER OF CHILDREN ON WHICH EACH IS BASED, THE STANDARD DEVIATIONS, COEFFICIENTS OF VARIABILITY AND THE RELIABILITIES OF THESE CONSTANTS

Age	N	Mean mm.	PE	Standard Deviation	PE	Coefficient Variability	PE
3 Months	116	592.85	1.35	21.57	.96	3.64	.16
6 "	132	655.74	1.33	22.67	.94	3.46	.17
9 "	141	702.54	1.49	26.17	1.05	3.73	.15
12 "	148	742.49	1.58	28.51	1.12	3.84	.15
18 "	149	805.98	1.69	30.63	1.20	3.80	.15
2 Years	165	861.59	1.66	31.53	1.17	3.66	.14
2½ "	170	911.24	1.67	32.36	1.18	3.55	.13
3 "	183	954.70	1.70	34.18	1.21	3.58	.13
3½ "	182	995.01	1.77	35.37	1.25	3.55	.13
4 "	179	1031.71	1.88	37.25	1.33	3.61	.13
4½ "	186	1069.87	2.07	41.84	1.46	3.91	.14
5 "	172	1103.48	2.15	41.75	1.52	3.78	.14
6 "	121	1174.12	2.74	44.65	1.94	3.80	.17
7 "	114	1232.14	2.92	45.82	2.11	3.72	.17
8 "	73	1292.95	3.85	48.81	2.72	3.78	.21
9 "	66	1356.61	4.23	50.94	2.99	3.75	.22
10 "	88	1407.50	3.97	55.19	2.81	3.92	.20
11 "	101	1476.49	4.39	65.35	3.10	4.43	.21
12 "	88	1552.96	4.82	66.98	3.41	4.31	.22
13 "	48	1598.54	5.34	54.87	3.78	3.43	.24

TABLE VII. INQUIRY STANDARDS FEMALE WEIGHT. THE MEANS WITH THE NUMBER OF CHILDREN ON WHICH EACH IS BASED, THE STANDARD DEVIATIONS, COEFFICIENTS OF VARIABILITY AND THE RELIABILITIES OF THESE CONSTANTS

Age	N	Mean Lbs.	PE	Standard Deviation	PE	Coefficient Variability	PE
3 Months	116	13.09	.12	1.31	.08	10.00	.45
6 "	132	17.03	.10	1.73	.07	10.16	.43
9 "	141	19.65	.12	2.13	.09	10.84	.44
12 "	148	21.91	.15	2.61	.10	11.91	.47
18 "	152	25.02	.17	3.03	.12	12.12	.48
2 Years	165	27.51	.16	3.07	.11	11.14	.42
2½ "	171	30.07	.18	3.53	.13	11.75	.43
3 "	184	32.49	.15	3.11	.11	9.56	.34
3½ "	182	34.85	.22	4.34	.15	12.44	.45
4 "	179	37.12	.23	4.63	.17	12.47	.45
4½ "	186	40.37	.25	5.06	.18	12.54	.45
5 "	172	42.34	.33	6.30	.23	14.89	.55
6 "	121	48.55	.41	6.74	.29	13.89	.61
7 "	114	53.99	.47	7.44	.33	13.78	.63
8 "	73	61.45	.72	9.11	.51	14.82	.85
9 "	66	70.92	1.02	12.30	.72	17.34	1.05
10 "	88	77.60	.92	12.74	.65	16.41	.86
11 "	101	87.04	.98	14.66	.70	16.85	.82
12 "	88	102.66	1.18	16.39	.83	15.96	.83
13 "	48	114.58	1.54	15.81	1.09	13.79	.97

All stature and weight data were reduced in December, 1935 and the differences between means of that date and those of July, 1937 are found in Table VIII. Over the age range 3 months to 7 years, the differences in male stature range from 0.38 mm. to 6.42 mm. with a mean difference of 2.31 mm.; the differences in male weight range from 0.02 pounds to 1.04 pounds with a mean difference of 0.29 pounds. The differences in female stature range from 0.04 mm. to 6.63 mm. with a mean difference of 1.96 mm.; the differences in female weight range from 0.00 to 0.67 pounds with a mean of 0.19 pounds. None of the differences is significant. The greatest stature difference, 6.63 mm., between the means of female stature at five years, shows a critical ratio of 1.37; the greatest

TABLE VIII. DIFFERENCES BETWEEN MEANS OF DECEMBER, 1935 AND OF JULY, 1937, MALE AND FEMALE, STATURE AND WEIGHT

	Diff. in Number MALE	Diff. in Stature mm. MALE	CR	Diff. in Weight lbs. MALE	CR	Diff. in Number FEMALE	Diff. in Stature mm. FEMALE	CR	Diff. in Weight lbs. FEMALE	CR
3 Months	13	2.22	.90	0.24	1.35	2	0.04	.01	0.26	1.50
6 "	15	.61	.23	.19	.89	7	.96	.34	.31	1.44
9 "	15	.38	.14	.31	1.17	11	1.08	.34	.27	1.03
12 "	14	.90	.29	.11	.39	14	1.15	.34	.00	.00
18 "	16	1.90	.54	.27	.87	11	1.17	.33	.00	.00
2 Years	22	1.47	.39	.02	.06	15	4.72	1.34	.12	.35
2½ "	21	1.33	.35	.04	.10	12	.98	.28	.05	.13
3 "	23	1.01	.25	.07	.18	15	.84	.23	.20	.54
3½ "	27	.89	.21	.23	.51	27	3.77	.94	.09	.19
4 "	47	1.60	.36	.12	.29	47	.60	.14	.15	.28
4½ "	58	6.42	1.27	.42	.80	61	1.97	.42	.15	.26
5 "	42	6.13	1.37	.10	.17	45	6.63	1.37	.11	.17
6 "	29	3.34	.54	1.04	1.18	28	1.96	.32	.33	.38
7 "	28	4.18	.57	.84	1.03	45	1.58	.22	.67	.58

weight difference, 1.04 pounds, between the means of male weight at six years, shows a critical ratio of 1.18.

To determine the statistical significance of the time interval between our successive examinations critical ratios of the age differences have been calculated in male and female stature and weight. These appear in Table IX. In stature, these critical ratios range from 59.27 to 4.27. The critical ratios for male stature decrease regularly in value over the entire age range and nowhere is the differ-

TABLE IX. SIGNIFICANCE OF THE DIFFERENCE BETWEEN MEANS AT SUCCESSIVE AGE LEVELS, MALE AND FEMALE, STATURE AND WEIGHT

	CR Male Stature	CR Female Stature	CR Male Weight	CR Female Weight
3 months–12 months	59.27	48.54	41.02	35.77
12 months– 2 years	33.91	35.09	16.76	17.45
2 years – 3 years	23.80	26.43	13.34	15.05
3 " – 4 "	19.51	20.48	11.02	11.16
4 " – 5 "	15.87	16.97	8.83	8.82
5 " – 6 "	12.07	13.69	7.76	7.97
6 " – 7 "	10.54	9.77	7.04	5.86
7 " – 8 "	8.54	8.48	5.22	5.86
8 " – 9 "	7.46	7.51	3.93	5.11
9 " –10 "	6.96	5.92	5.25	3.29
10 " –11 "	6.13	7.87	3.72	4.74
11 " –12 "	4.78	7.67	2.57	6.86
12 " –13 "	4.77	4.27	3.62	4.15
3 months– 6 months	25.50	22.37	21.60	20.35
6 " – 9 "	17.99	15.82	12.99	11.18
9 " –12 "	14.85	12.42	8.12	8.08
12 months–18 months	18.83	18.49	10.62	9.53
18 months– 2 years	14.70	15.84	6.67	7.26
2 years – 2½ "	13.30	14.22	6.32	7.10
2½ " – 3 "	10.99	12.27	6.19	6.83
3 " – 3½ "	10.23	11.07	6.02	5.98
3½ " – 4 "	9.31	9.60	5.29	4.81
4 " – 4½ "	8.38	9.21	4.50	6.41
4½ " – 5 "	7.75	7.60	5.08	3.24

ence between two successive examination ages insignificant. The critical ratios for female stature follow the same pattern up to the 10–11 year difference where there is an increase in critical ratio which is retained in the 11–12 year difference. In weight the critical ratios range from 41.02 to 2.57, all surpassing in value the criterion of significance (CR = 3.0) with the exception of the one mentioned (2.57). It is evident then that nowhere in our range have we chosen too short an interval between examination dates for the reduction of age-typical constants.

THE CONSISTENCY (SELF-CORRELATION) OF WEIGHT AND STATURE MEASUREMENTS

In the absence of repetition of measurements on the same child, at the same date, by the same observer we are unable to present reliability coefficients for the measurements under discussion. That remeasurement is impracticable is

TABLE X. CORRELATION COEFFICIENTS BETWEEN STATURE AT ONE EXAMINATION AGE AND THAT OF THE FOLLOWING YEAR

	1 yr.	2 yr.	3 yr.	4 yr.	5 yr.	6 yr.	7 yr.	8 yr.	9 yr.	10 yr.	11 yr.	12 yr.	13 yr.
3 mos.—M.	659												
F.	723												
1 yr.— M.		877											
F.		830											
2 yr.— M.			912										
F.			921										
3 yr.— M.				913									
F.				952									
4 yr.— M.					946								
F.					970								
5 yr.— M.						970							
F.						971							
6 yr.— M.							969						
F.							974						
7 yr.— M.								978					
F.								973					
8 yr.— M.									976				
F.									963				
9 yr.— M.										982			
F.										968			
10 yr.— M.											984		
F.											929		
11 yr.— M.												972	
F.												971	
12 yr.— M.													968
F.													963

TABLE XI. CORRELATION COEFFICIENTS BETWEEN WEIGHT AT ONE EXAMINATION AGE AND THAT OF THE FOLLOWING YEAR

	1 yr.	2 yr.	3 yr.	4 yr.	5 yr.	6 yr.	7 yr.	8 yr.	9 yr.	10 yr.	11 yr.	12 yr.	13 yr.
3 mos.—M.	608												
F.	639												
1 yr.— M.		884											
F.		828											
2 yr.— M.			912										
F.			898										
3 yr.— M.				942									
F.				936									
4 yr.— M.					958								
F.					935								
5 yr.— M.						915							
F.						862							
6 yr.— M.							922						
F.							954						
7 yr.— M.								933					
F.								883					
8 yr.— M.									931				
F.									929				
9 yr.— M.										946			
F.										923			
10 yr.— M.											968		
F.											929		
11 yr.— M.												967	
F.												940	
12 yr.— M.													955
F.													909

evident when one considers the extent of the regular serial examination and the time required for its components, anthropometric, roentgenographic, and psychometric. We are therefore offering, as a partial solution to this problem, age-with-age correlation coefficients. The problem of consistency of growth also enters into these coefficients. The age-with-age correlation coefficient is then not solely an indication of the reliability of the measurements of a dimension or solely an indication of the consistency of rank in a measurement, such as stature or weight, from age to age. Both of these factors are represented by the correlation coefficient and their proportionate representation is necessarily unknown. With this understanding the age-with-age correlation coefficient can be

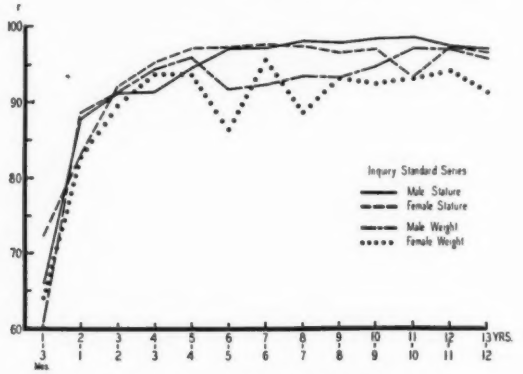

FIG. 9. Curves of correlation coefficients of stature with stature and weight with weight at regular examinations made at yearly intervals. (See Tables X, XI.)

interpreted as indicating both high reliability of the measurement and high consistency in rank from year to year so long as the correlation coefficient has a high forecasting efficiency. When a correlation coefficient has a relatively low forecasting efficiency it must be interpreted merely as an indication that the measure at one age cannot be predicted efficiently from the measure at the previous age. The correlation coefficient itself gives us no information on whether the cause is to be sought in widely differentiated growth of individuals or unreliable measurements or a combination of both.

In Tables X and XI and Figure 9 are presented the correlation coefficients measuring the relationship of stature to stature at successive ages and the relationship of weight to weight at successive ages. Male and female stature correlations progress from low values of 0.723 and 0.659, respectively, for the interval between three and twelve months to high values of 0.970 and 0.971 for the interval between five and six years; they remain high thereafter. The weight correlations rise from 0.608 (male) and 0.639 (female) at the interval between three and twelve months, to 0.915 and 0.862 respectively at the interval between five and six years, and thereafter they remain between 0.88 and 0.97, the male tend-

ing to be higher than the female. The weight correlation coefficients tend to be somewhat lower and less regular than the stature coefficients.

Stature, as an instrument of prediction, is extremely satisfactory, weight only a little less so. From our data it can be concluded that after the second birthday until the thirteenth a child's rank in the group stature distribution is very stable and consistent from year to year. It will be shown that we have no child of more

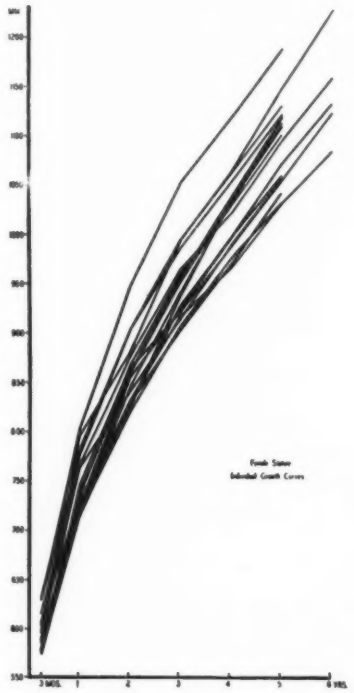

FIG. 10. Individual curves recording growth in stature from three months to six years (girls).

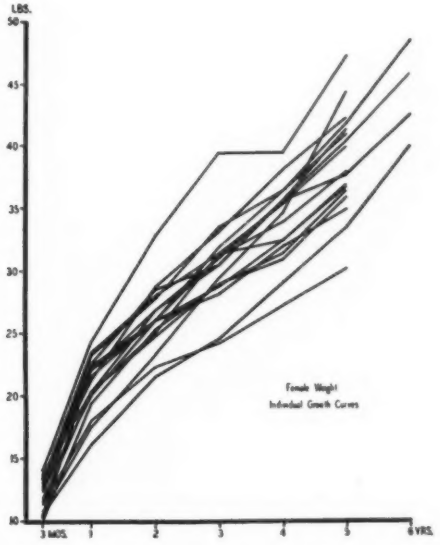

FIG. 11. Individual curves recording growth in weight from three months to six years (girls).

than two years of age whose rank in the stature distribution changes from one extreme to the other whereas over the interval three months to two years there are a few children whose stature rank does shift through the majority of that range. Their great change in rank naturally affects the age-with-age correlation coefficients; the correlations between three and twleve months, and between one and two years of age, are lower than the ones thereafter. Bayley and Davis report a stature correlation of 0.93 and a weight correlation of 0.91 between one and two years of age, when both sexes were combined into a small group. Figures 10 and 11 represent individual curves in stature and weight in girls and

illustrate further this growth consistency. Only a few of the curves showing practical regularity of increment have been included but none of the really irregular curves has been omitted. In Figures 12 and 13 are curves illustrating the progress of stature and weight in 32 boys from different sections of the age range, three months to eleven years. These typify the homogeneity of our population at different ages.

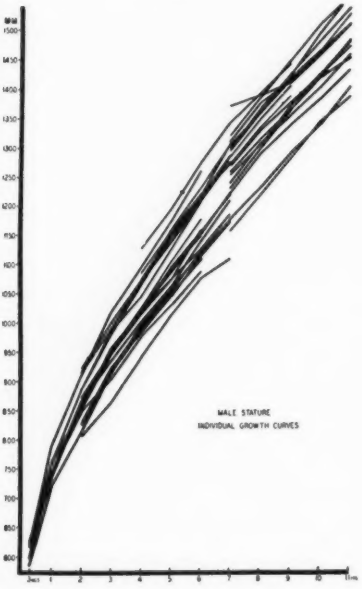

FIG. 12. Individual curves recording growth in stature of 32 boys from different sections of the age range.

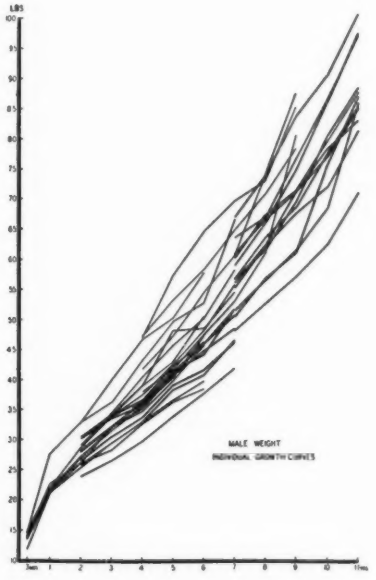

FIG. 13. Individual curves recording growth in weight of 32 boys from different sections of the age range.

A part of the general problem of consistency in stature rank over a period of time is the question of the difference in consistency between upper and lower extremes of the stature distribution. It is pertinent therefore to inquire whether the shortest children are more likely to retain their rank than the tallest children, or whether there is more fluctuation between the shortest twenty five percent and the middle fifty percent than there is between the tallest twenty five percent and the middle fifty percent of the distribution. That there is some shifting in rank we know. We know, also, that after two years of age, there is no shifting through the majority of the range. To attempt to answer this question longitudinal data would be necessary and will be available for our entire age period on completion of our study. For the present we have divided our measurements from three months to eight years of age into four longitudinal sections of two years each; these are represented in Figures 14 and 15. We used the sim-

ple procedure of sorting out the upper and lower twenty five percent of the stature distribution at one age, repeating this process and using the same sampling at a later age in order to obtain the proportion of children who appear in both upper divisions and the proportion of children who appear in both lower divisions. Our age divisions are three months to two years, two to four years, four to six years and six to eight years. Figures 14 (male) and 15 (female) are

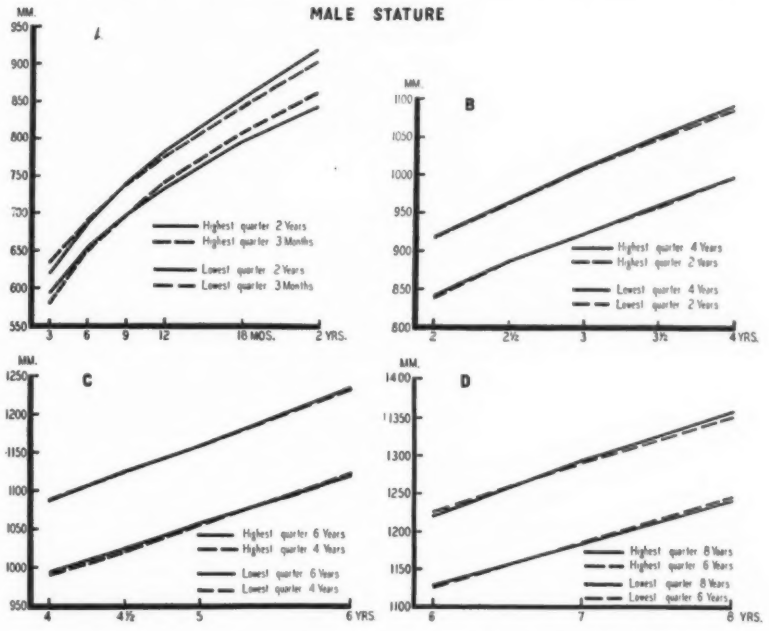

FIG. 14. Curves of the means of the samples of boys who constitute the highest and the lowest quarters of the distribution at the beginning or at the end, respectively, of the age period represented.

graphic representations of the age means of all four groups in each age section.

We have a series of ninety nine males with complete records from three months to two years. From this series, two small groups of approximately twenty five percent of the total were sorted, the twenty five longest and the twenty five shortest boys at three months of age. Means of horizontal length in each of these two groups at every examination from three to twelve months and means of stature at eighteen months and two years are plotted and appear in the solid lines of Figure 14 A. From the same group of ninety nine males we then sorted the tallest and shortest twenty five percent at two years; the means of these two groups appear in the dotted lines of Figure 14 A. From these four groups of boys, the number common to both upper divisions and the number common to both lower divisions were then counted and the percentage of their

group obtained. Thirteen of twenty five or fifty two percent of the boys longest at three months are also tallest at two years; thirteen of twenty five or fifty two percent of the boys shortest at three months are also shortest at two years. Four of the twenty five shortest at three months shift to the tallest division at two years while two of the twenty five longest at three months shift to the shortest division at two years. In this period covering the first two years of life alone among the age divisions which we have selected do these extreme shifts occur.

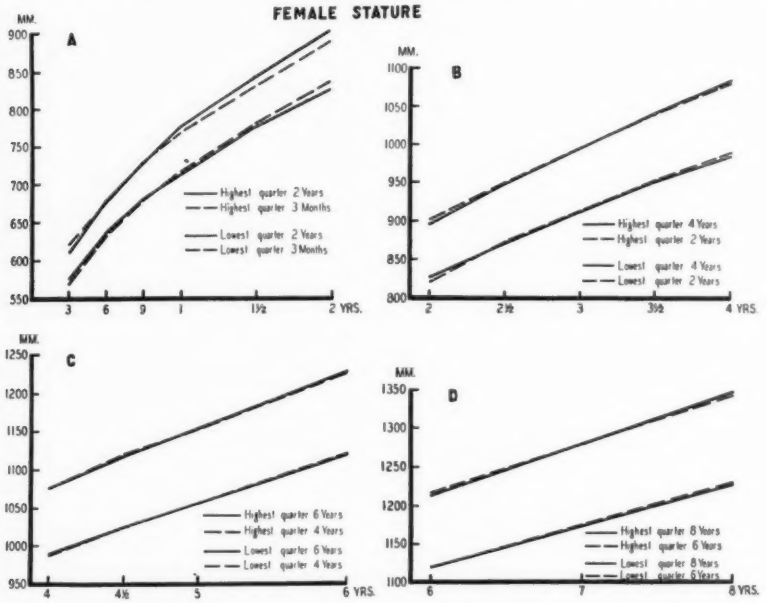

FIG. 15. Curves of the means of the samples of girls who constitute the highest and the lowest quarters of the distribution at the beginning or at the end, respectively, of the age period represented.

The correlation between horizontal length at three months and stature at two years, computed from the measurements of 101 boys, is 0.609.

There are 108 boys in our age division from two years to four years who have complete records. In Figure 14 B are the curves of the mean statures over this period of the boys comprising the upper and lower quarters of the stature distribution at two years and of the boys comprising the upper and lower quarters at four years. Of the boys who are shortest at two years, twenty four of twenty seven or 89 percent are also shortest at four years; of those tallest at two years, twenty three of twenty seven or 85 percent are tallest at four years. The correlation coefficient between stature at two years and stature at four years based upon 114 boys is 0.914.

Sixty boys have complete records from four years to six years. In Figure 14 C

TABLE XII. TABULAR PRESENTATION OF DATA UTILIZED IN FIGURES 14, 15

Stature 3 mo.–2 yrs.	Mean 3 mo.	Mean 6 mo.	Mean 9 mo.	Mean 12 mo.	Mean 18 mo.	Mean 2 yrs.	Children in Both Highest Quarter No.	%	Children in Both Lowest Quarter No.	%	Children Shifting Low to High No.	%	Children Shifting High to Low No.	%
Male = 99														
Highest 25%—3 mo.	633	690	737	774	838	896	13/25	52					2/25	8
Highest 25%—2 yrs.	620	686	740	782	849	911	13/25	52			4/25	16		
Lowest 25%—3 mo	582	650	698	740	803	855			13/25	52	4/25	16		
Lowest 25%—2 yrs.	593	655	697	732	793	836			13/25	52			2/25	8
Female = 92														
Highest 25%—3 mo.	618	675	728	767	830	888	13/23	57					2/23	9
Highest 25%—2 yrs.	609	676	728	775	838	900	13/23	57			0	0		
Lowest 25%—3 mo.	567	631	677	715	778	832			15/23	65	0	0		
Lowest 25%—2 yrs.	574	636	679	713	775	824			15/23	65			2/23	9

2 yrs.–4 yrs. Male = 108	Mean 2 yrs.	Mean 2½ yrs.	Mean 3 yrs.	Mean 3½ yrs.	Mean 4 yrs.	Children in Both Highest Quarter No.	%	Children in Both Lowest Quarter No.	%	Children Shifting Low to High No.	%	Children Shifting High to Low No.	%
Highest 25%—2 yrs.	916	961	1007	1045	1081	23/27	85					0	0
Highest 25%—4 yrs.	916	964	1009	1050	1087	23/27	85				0		
Lowest 25%—2 yrs.	837	884	921	957	994			24/27	89	0	0		
Lowest 25%—4 yrs.	840	884	921	958	993			24/27	89			0	0
Female = 121													
Highest 25%—2 yrs.	901	949	995	1039	1079	24/30	80					0	0
Highest 25%—4 yrs.	896	948	995	1040	1082	24/30	80			0	0		
Lowest 25%—2 yrs.	821	872	911	952	988			23/30	77	0	0		
Lowest 25%—4 yrs.	826	871	910	950	983			23/30	77			0	0

4 yrs.–6 yrs. Male = 60	Mean 4 yrs	Mean 4½ yrs.	Mean 5 yrs	Mean 6 yrs.	Children in Both Highest Quarter No.	%	Children in Both Lowest Quarter No.	%	Children Shifting Low to High No.	%	Children Shifting High to Low No.	%
Highest 25%—4 yrs.	1081	1119	1153	1228	12/15	80					0	0
Highest 25%—6 yrs.	1086	1125	1160	1235	12/15	80			0	0		
Lowest 25%—4 yrs.	990	1021	1057	1121			12/15	80	0	0		
Lowest 25%—6 yrs.	993	1024	1058	1119			12/15	80			0	0
Female = 82												
Highest 25%—4 yrs.	1075	1118	1153	1227	18/20	90					0	0
Highest 25%—6 yrs.	1075	1119	1154	1228	18/20	90			0	0		
Lowest 25%—4 yrs.	986	1024	1055	1121			18/20	90	0	0		
Lowest 25%—6 yrs.	987	1024	1055	1119			18/20	90			0	0

6 yrs.–8 yrs. Male = 64	Mean 6 yrs.	Mean 7 yrs.	Mean 8 yrs.	Children in Both Highest Quarter No.	%	Children in Both Lowest Quarter No.	%	Children Shifting Low to High No.	%	Children Shifting High to Low No.	%
Highest 25%—6 yrs.	1225	1290	1351	11/16	69					0	0
Highest 25%—8 yrs.	1221	1293	1357	11/16	69			0	0		
Lowest 25%—6 yrs.	1123	1184	1245			13/16	81	0	0		
Lowest 25%—8 yrs.	1126	1183	1241			13/16	81			0	0
Female = 64											
Highest 25%—6 yrs.	1216	1280	1342	13/16	81					0	0
Highest 25%—8 yrs.	1212	1280	1346	13/16	81			0	0		
Lowest 25%—6 yrs.	1119	1175	1228			15/16	94	0	0		
Lowest 25%—8 yrs.	1119	1174	1227			15/16	94			0	0

	N	r 3 mo.–2 yrs. Stature	N	r 2–4 yrs. Stature	N	r 4–6 yrs. Stature	N	r 6–8 yrs. Stature	N	r 10–12 yrs. Stature	N	r 2–5 yrs. Stature	N	r 5–8 yrs. Stature
Male	101	0.609	114	0.914	66	0.940	65	0.938	35	0.944	88	0.893	49	0.893
Female	98	0.718	130	0.860	85	0.964	64	0.955	27	0.918	89	0.775	59	0.915
	N	r 3 mo.–2 yrs. Weight	N	r 2–4 yrs. Weight	N	r 4–6 yrs. Weight	N	r 6–8 yrs. Weight	N	r 10–12 yrs. Weight	N	r 2–5 yrs. Weight	N	r 5–8 yrs. Weight
Male	101	0.591	116	0.880	66	0.906	65	0.851	35	0.915	89	0.859	49	0.686
Female	98	0.560	130	0.852	85	0.886	64	0.917	27	0.822	89	0.749	59	0.716

are the curves of mean stature of the upper and lower quarters at four years and the upper and lower quarters at six years. Twelve of fifteen or 80 percent of the boys who are shortest at four years are shortest also at six years; twelve of fifteen or 80 percent of those tallest at four years are also tallest at six years.

The correlation coefficient between stature at four years and stature at six years based upon 66 boys is 0.940.

Sixty four boys have complete records from six years to eight years. In Figure 14 D are curves of the mean stature of the upper and lower quarters at six years and the upper and lower quarters at eight years. Thirteen of sixteen or 81 percent of the boys who are shortest at six years are also shortest at eight years; twelve of sixteen or 69 percent of those tallest at six years are also tallest at eight years. The correlation coefficient between stature at six years and stature at eight years based upon 65 boys is 0.938.

TABLE XIII. CRITICAL RATIOS OF THE SEX DIFFERENCE IN STATURE AND WEIGHT FROM 3 MONTHS TO 13 YEARS

Stature		Weight	
Age	CR	Age	CR
3 months	8.61	3 months	7.50
12 "	5.97	12 "	6.65
2 years	3.59	2 years	4.56
3 "	2.06	3 "	3.05
4 "	1.96	4 "	2.55
5 "	1.30	5 "	1.30
6 "	0.30	6 "	0.02
7 "	1.05	7 "	0.69
8 "	1.06	8 "	0.54
9 "	0.38	9 "	0.75
10 "	0.79	10 "	0.49
11 "	1.23	11 "	0.25
12 "	4.19	12 "	4.05
13 "	2.55	13 "	3.73
6 months	6.04	6 months	6.99
9 "	6.00	9 "	7.68
18 months	4.65	18 months	5.97
2½ years	3.07	2½ years	3.23
3½ "	1.76	3½ "	2.43
4½ "	1.63	4½ "	0.61

Obviously, these percentages, themselves, are not completely reliable because of the small number of children upon whose measurements they are based. Evident, also, however, is the fact that there is little difference in the consistency of the upper and lower extremes of the stature distribution from year to year. Figure 15 A, B, C, D, graphically represents corresponding data based upon the measurements of girls. The same consistency of the upper and lower quarters of the stature distribution occurs here. (See Table XII.)

From the assembled records of both boys and girls, we may conclude therefore that the general measure of relationship, the correlation coefficient, is equally efficient in representing the tall groups, the short groups and the undifferentiated middle groups of our population.

SEX DIFFERENCES IN STATURE AND WEIGHT

It is customary to use standards of stature and weight appropriate to each sex, although over a period of some eight years in the child's growing life differences between boys and girls in these two measures are, in our experience, sta-

tistically insignificant. The significance of the sex difference in stature and weight at the examination dates from three months to thirteen years computed from our data is shown in Table XIII and Figure 16 C. Our boys and girls are

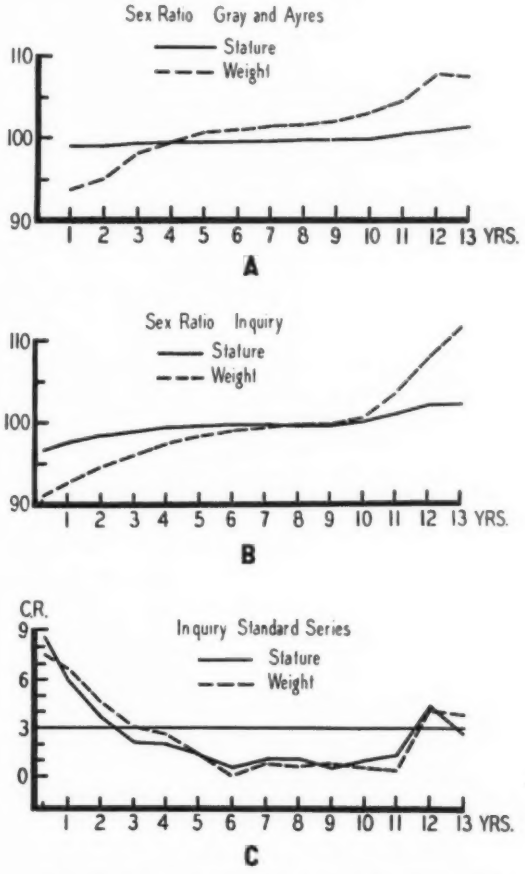

FIG. 16. A. Curves representing sex ratios mean female /mean male for stature and weight. Data from Gray and Ayres. B. Curves representing sex ratios mean female/mean male for stature and weight. Inquiry data. C. Curves representing critical ratios of sex differences for stature and weight. Inquiry data.

not significantly different in stature from three to eleven years, inclusive, or in weight from four to eleven years, inclusive. There are differences in stature and weight between boys and girls within these age ranges; they are individual in nature and not sex-linked. We have employed the usual critical ratio of three

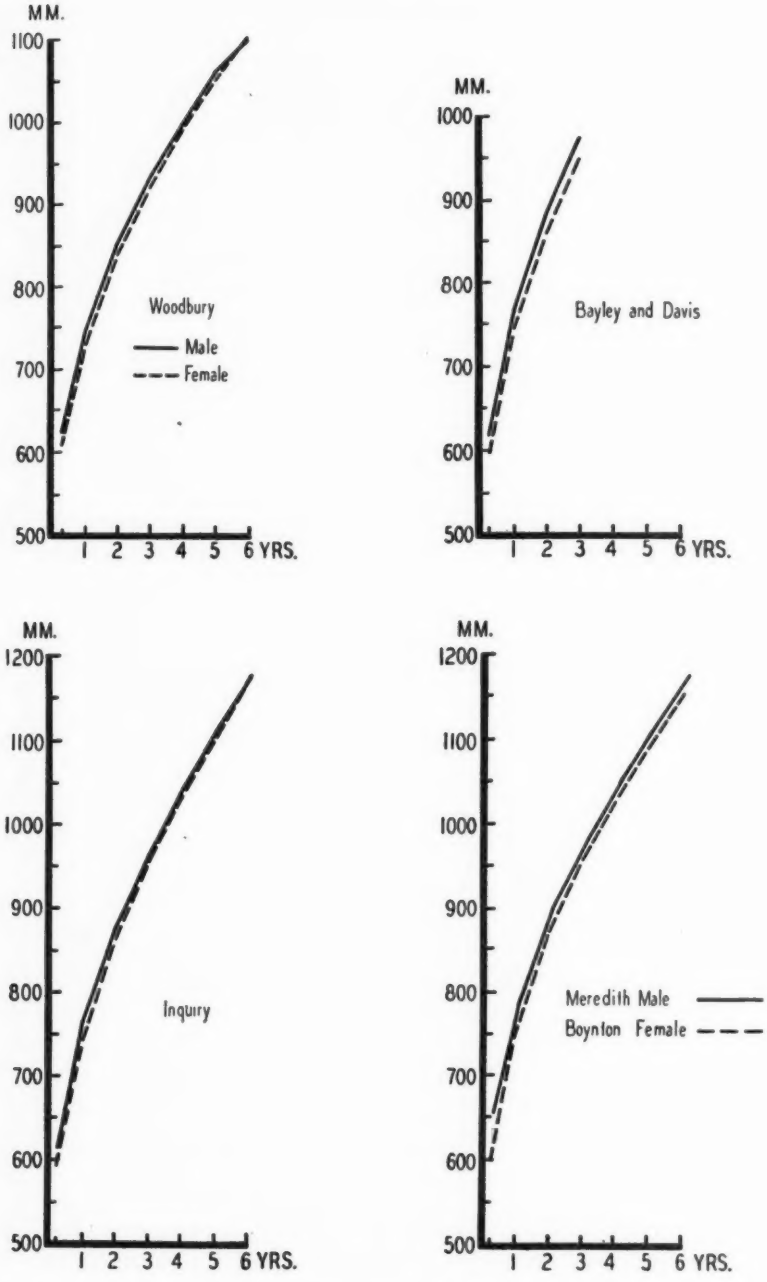

FIG. 17. Comparison of stature curves, male and female, from published samplings three months to six years.

as a test for significance rather than Fisher's tables of "t" which postulate the use of a scale of critical ratio values dependent upon the number of degrees of freedom. The period of life during which sex differences are negligible thus corresponds to that period when differences in annual increments of stature and weight in both sexes are also very small.

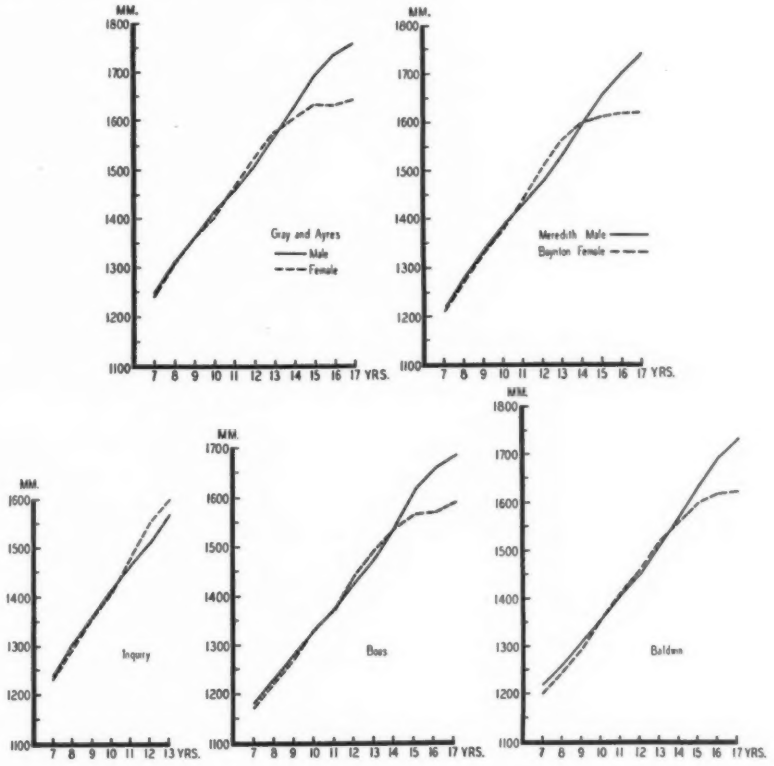

FIG. 18. Comparison of stature curves, male and female, from published samplings, seven to seventeen years.

Following the example of Gray and Ayres we have employed the sex ratio, $\frac{\text{Mean Female}}{\text{Mean Male}}$ for purposes of comparison. We have plotted this ratio for both stature and weight over our age range, three months to thirteen years. These curves have been smoothed for easier comparison with Gray and Ayres' smoothed curves which are reproduced in Figure 16 A. The stature curves of the two populations are very similar, the only outstanding differences being the greater sex disparity of our group during the first two years and during the twelfth and thirteenth years. No one of these differences is pronounced. The

two curves of weight ratios, however, show greater difference particularly between three and eleven years, inclusive, the Gray and Ayres' ratios being consistently higher than ours over this period. Reference to Figures 4 and 6 (comparative curves of annual status) will show that Gray and Ayres' girls are heavier than our girls while the two groups of boys are very similar in weight. Also, the evident irregularity of our twelve and thirteen year curves is due to the inclusion of several unusually heavy girls, the records of whom we have hesitated to eliminate because of their obvious good health.

The series of stature curves shown in Figures 17 and 18 present more similarities than differences particularly below the age of six. Nevertheless the diversity of ages at which the curves show stature in girls to be greater than stature in boys and the amount of this difference may be variously interpreted. These differences, however, must be functions of the samplings upon which they are based. It is therefore possible that no one of these examples can be used as anything more than a partial description of its particular population—that no one can be safely used as a completely adequate foundation for generalizations on sex differences in stature.

From our data, we find that our mean female stature is somewhat greater than our mean male stature at six and again at eleven, twelve and thirteen years of age. The slight female superiority shown in our curves at six years will probably disappear when more girls from our younger group will be available for our six year standards. Our mean female weight is superior to our mean male weight at age nine and again at ages eleven, twelve and thirteen. The mean female weight at nine years is computed from an inadequate number of measurements and this superiority, already insignificant, may well disappear when the number of our nine year old girls increases. Gray and Ayres' girls are insignificantly superior to their boys at age nine in stature. The Gray and Ayres' data show a female superiority in stature at eleven, twelve and thirteen years; Boynton's mean female stature is superior to Meredith's mean male stature at eleven, twelve, thirteen and fourteen years; Baldwin's mean female stature is slightly superior to his mean male stature at eleven, twelve and thirteen years, and Boas' mean female stature is superior to his mean male stature at ten, twelve and thirteen years of age.

There is, therefore, very definite evidence of a trend which may begin in the statistically insignificant superiority of female stature and weight at about the age of nine years but progresses in degree until it becomes statistically significant around twelve years.

MEANS, VARIABILITIES AND RELIABILITIES OF ANNUAL INCREMENTS IN STATURE AND WEIGHT

In Tables XIV and XV are presented the means, standard deviations, coefficients of variability and their reliabilities, that is, the constants of annual increments in stature and weight over the three month to thirteen year period. Although we have as yet a relatively small number of increment measurements above five years of age, Figures 19 and 20, comparative curves of increment in

male and female stature and weight, show that our curves of both stature and weight are reasonably smooth and free from evidences of "spurts and slumps." In these figures we have used our own computed increments; in plotting curves of other investigators we have calculated increments from differences between successive age means. All the curves shown are unsmoothed.

It is reasonable to conclude that the relative absence of age to age irregularities in the Inquiry increment curves is largely due to the fact that children of

TABLE XIV. MEANS, VARIABILITIES AND RELIABILITIES OF INCREMENT IN STATURE (MALES, FEMALES)

Age	N	Mean mm.	PE	Standard Deviation	PE	Coefficient Variability	PE
Stature—Male							
3 to 12 Months	101	150.36	1.22	18.16	.86	12.08	.58
1 to 2 Years	49	123.92	1.20	12.49	.85	10.08	.69
2 to 3 "	129	88.06	1.18	19.90	.84	22.60	1.00
3 to 4 "	127	73.92	.59	9.86	.42	13.33	.57
4 to 5 "	121	69.41	.65	10.54	.46	15.19	.67
5 to 6 "	84	67.02	.76	10.27	.53	15.33	.82
6 to 7 "	84	64.12	.95	12.93	.67	20.16	1.09
7 to 8 "	75	61.20	.75	9.69	.53	15.84	.89
8 to 9 "	64	55.67	.91	10.83	.65	19.45	1.20
9 to 10 "	52	54.88	.93	9.98	.66	18.18	1.24
10 to 11 "	65	51.85	.70	8.34	.49	16.08	.98
11 to 12 "	62	53.21	1.26	14.71	.89	27.64	1.80
12 to 13 "	61	60.95	1.61	18.67	1.14	30.64	2.04
Stature—Female							
3 to 12 Months	87	150.05	1.37	18.91	.97	12.60	.65
1 to 2 Years	44	131.98	1.84	18.10	1.30	13.71	1.00
2 to 3 "	121	90.03	.87	14.19	.62	15.76	.70
3 to 4 "	142	79.05	.63	11.10	.44	14.05	.57
4 to 5 "	137	72.20	.58	10.12	.41	14.02	.58
5 to 6 "	98	68.03	.66	9.62	.46	14.14	.69
6 to 7 "	92	62.60	.73	10.36	.51	16.54	.84
7 to 8 "	69	57.78	.83	10.19	.59	17.64	1.04
8 to 9 "	52	60.13	1.18	12.62	.84	20.99	1.45
9 to 10 "	35	57.69	1.48	12.95	1.13	22.45	1.90
10 to 11 "	60	61.32	1.57	18.03	1.11	29.40	1.96
11 to 12 "	56	66.86	1.46	16.19	1.03	24.21	1.63
12 to 13 "	41	55.12	2.07	19.62	1.46	35.59	2.97

the Inquiry series are always measured within seven days of the regular, specified examination dates. It is assumed that large numbers of children accepted for examination over the whole year and classified as of the age of their nearest birthdays will present a balanced age distribution. In actual practice this cannot always occur. The mean age of children classified as five years old may well be four years, eleven and a half months, whereas the mean age of the six year old classification of the same series may be six years and one half month. In this event, the five year to six year increment would be a thirteen month increment. If in the same series the mean age of the seven year group happened to be six years, eleven and one half months, the six to seven year increment would be

only an eleven month increment. This possible sequence of a thirteen month increment and an eleven month increment would cause a zig-zag appearance of the increment curve of that series and may indeed have occurred in some of the data here discussed.

In our data, stature shows a continuous decrease in mean annual increment from birth to eleven years in the female and to twelve years in the male when a reversal occurs and the mean annual increment increases. The period of so called

TABLE XV. MEANS, VARIABILITIES AND RELIABILITIES OF INCREMENT IN WEIGHT (MALES, FEMALES)

Age	N	Mean lbs.	PE	Standard Deviation	PE	Coefficient Variability	PE
Weight—Male							
3 to 12 Months	101	9.32	.14	2.04	.10	21.91	1.09
1 to 2 Years	119	4.97	.09	1.45	.06	29.23	1.38
2 to 3 "	129	4.01	.08	1.41	.06	35.17	1.65
3 to 4 "	127	4.13	.09	1.66	.07	40.09	1.95
4 to 5 "	121	4.60	.10	1.58	.07	34.48	1.66
5 to 6 "	84	4.51	.17	2.34	.12	51.82	3.34
6 to 7 "	84	5.57	.20	2.76	.14	49.48	3.14
7 to 8 "	75	6.70	.29	3.74	.21	55.81	3.92
8 to 9 "	64	6.64	.29	3.41	.20	51.35	3.78
9 to 10 "	52	7.92	.38	4.06	.27	51.19	4.18
10 to 11 "	65	8.81	.39	4.67	.28	53.04	3.92
11 to 12 "	62	9.54	.44	5.11	.31	53.58	4.07
12 to 13 "	61	10.90	.49	5.72	.35	52.49	3.99
Weight—Female							
3 to 12 Months	87	8.07	.12	1.61	.08	19.92	1.06
1 to 2 Years	113	5.56	.11	1.72	.07	30.86	1.51
2 to 3 "	121	4.18	.10	1.61	.07	38.53	1.90
3 to 4 "	142	4.46	.10	1.81	.07	40.54	1.87
4 to 5 "	137	4.55	.12	2.07	.08	45.51	2.21
5 to 6 "	98	5.08	.20	2.95	.14	58.16	3.63
6 to 7 "	92	5.40	.17	2.47	.12	45.79	2.71
7 to 8 "	69	6.65	.35	4.34	.25	65.34	5.11
8 to 9 "	52	7.38	.41	4.41	.29	59.68	5.16
9 to 10 "	35	8.12	.53	4.62	.40	56.85	5.88
10 to 11 "	60	9.58	.40	4.56	.28	47.60	3.47
11 to 12 "	56	12.98	.49	5.43	.35	41.84	3.10
12 to 13 "	41	10.29	.62	5.91	.44	57.42	5.51

accelerated growth during the first two years of life, the "circumnatal growth spurt," should more properly be termed the postnatal period of rapidly decreasing increment. The decrease is much less during the second year and still less during the third but continues until the eleventh or twelfth year according to sex, the reduction in velocity being such that the decrease in annual increment is just measurable. It is apparent, then, that the stature of our children, both boys and girls, advances after birth at a continuously decreasing rate until after eleven (female) or twelve (male) when there is an increase in the yearly increment associated in time with approaching sex maturation.

The increment of weight during the first postnatal year is approximately twice

the birth weight. During the three-to-twelve month period, while increment in body length is undergoing its most rapid reduction, increment in weight exhibits its greatest increase. During the second year, weight increment is much less and does not equal in amount the birth weight. During the third year the decrease in weight increment is very slight and from the fourth year on there is a continuous, small but measurable, rise in annual weight increment. Thus, the weight of Inquiry children, both boys and girls, advances after birth at an increasing rate during the first year, a decreasing rate during the second and third

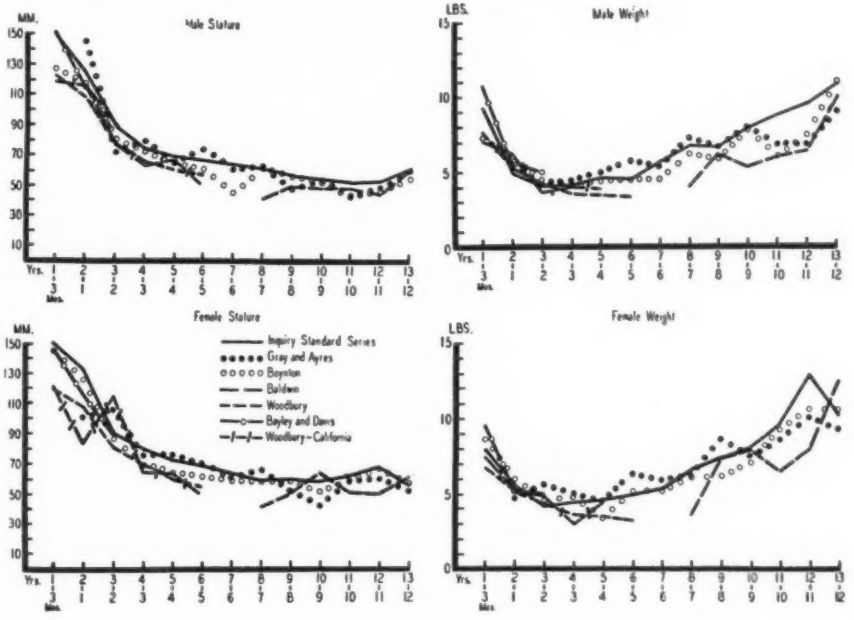

FIG. 19. Comparison of curves showing increment in stature, male and female, from published samplings.

FIG. 20. Comparison of curves showing increment in weight, male and female, from published samplings.

years and thereafter at a continuously increasing rate which rises still more during the years immediately preceding puberty.

Individual differences in annual increment are evident, the range for weight increments being greater than that for stature increments. There is, however, every reason to believe that healthy children whose increments fall in the lower division of the increment range will present the same type of growth curve as children whose increments fall in the higher division of the increment distribution, the only difference between the two being one of quantity.

There is an exception to this generalization in the behavior of the stature increment during the first year of life. Through the cooperation of Professor

A. H. Bill we have obtained records of the birth length and birth weight of children on the Inquiry series born at Maternity Hospital. The mean birth length of 70 males is 501 mm., of 47 females, 495 mm. The mean birth weight of 111 males is 7.55 pounds, of 103 females, 7.04 pounds. The weights were all recorded to the nearest gram; the lengths were recorded in inches or centimeters. (See Table XVI.)

TABLE XVI. MEANS AND VARIABILITIES OF BIRTH LENGTHS AND WEIGHTS

	N	Mean Birth Length mm.	SD	V	N	Mean Birth Weight lbs.	SD	V
Male	70	501.20	26.99	5.38	111	7.55	1.00	13.27
Female	47	495.04	23.37	4.72	103	7.04	1.07	15.19

The correlation coefficient between male horizontal length at birth and at three months (N = 58) we have found to be 0.557; between female horizontal lengths of the same ages (N = 38), 0.534. The correlation coefficient between male horizontal length at birth and at 12 months (N = 62) is 0.395; between female horizontal lengths of the same ages (N = 41), 0.299. The correlation coefficient between male weight at birth and at three months (N = 72) is 0.332; between female weights at the same ages (N = 56), 0.498. The correlation coefficient between male weights at birth and at twelve months (N = 86) is 0.324; between female weights at the same ages (N = 68), 0.485.

TABLE XVII. CORRELATION COEFFICIENTS BETWEEN STATUS AT ONE AGE LEVEL AND INCREMENT OVER THE FOLLOWING AGE INTERVAL COMPUTED FOR STATURE AND FOR WEIGHT

Status	Increment	N	Male Stature	N	Female Stature	N	Male Weight	N	Female Weight
Birth	Birth–one year	64	−462	41	−014	88	−127	68	141
3 months	3 months–1 year	104	−197	99	−046	106	166	101	161
1 year	1–2 years	61	198	57	055	119	317	114	044
2 years	2–3 "	129	083	119	110	132	141	124	249
3 "	3–4 "	123	033	137	126	125	398	138	363
4 "	4–5 "	114	259	134	277	114	336	134	377

Correlation coefficients between stature and weight status at one age level with the respective increment over the following interval are presented in Table XVII. Of particular interest are the negative coefficients between birth length and the birth to one year increment in length and of the three month length with the three month to one year increment in length. It appears that during the first postnatal year, our short babies gain in length more than our long babies and that this reversal occurs to a greater extent in the male than in the female.

Furthermore, the lowest quarter of the three month male horizontal length distribution makes a mean gain of 68 mm. from three to six months, 158 mm. from three to twelve months and 273 mm. from three months to two years. The highest quarter of the three month male horizontal length distribution makes a

mean gain of 57 mm. from three to six months, 141 mm. from three to twelve months and 263 mm. from three months to two years.

The lowest quarter of the three month female horizontal length distribution makes a mean gain of 64 mm. from three to six months, 148 mm. from three to twelve months and 265 mm. from three months to two years. The highest quar-

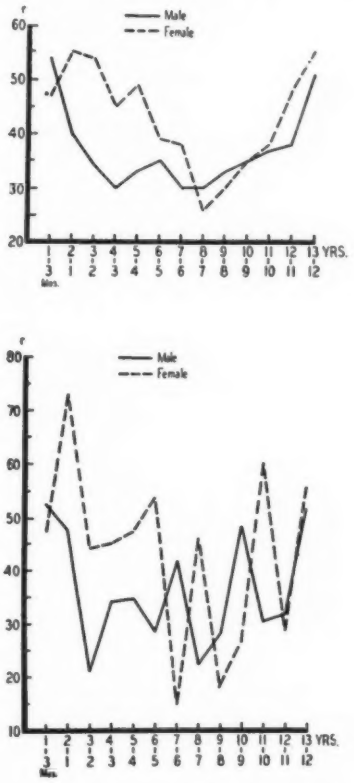

FIG. 21. Smoothed and unsmoothed curves representing correlation coefficients between annual increments in stature and weight (male and female).

ter of the three month female horizontal length distribution makes a mean gain of 57 mm. from three to six months, 149 mm. from three to twelve months and 270 mm. from three months to two years. It therefore appears that the more rapid gain in length and weight shown by the short babies in comparison with the long babies lasts through a greater period of the first twelve months in boys than it does in girls.

However, since the correlation coefficients between birth length and twelve

month length are positive, it is clear that the short babies do not make sufficient gain to overcome entirely their lower rank at birth. After one year of age the correlation coefficients between status at the age level and increment over the following year are all positive, although some approach zero and there is little apparent progression.

There is a positive relationship between annual increments in stature and in weight from one to thirteen years but the correlation coefficients are low and the curve of coefficients over the age range is very irregular (Figure 21 A). The smoothed curves of correlation coefficients for both sexes (Figure 21 B) show that the highest coefficients tend to be at the extremes, at the ages when absolute annual increment is highest. The range of the smoothed correlation coefficients falls between 0.26 and 0.55; the computed coefficients range from 0.147 to 0.730, both extremes occurring in the female distribution.

The healthy child's annual increment in stature and to a less extent in weight is not, then, an irregular, unpredictable, "slump and spurt" affair. We do not claim that the children upon whose measurements our conclusions are based are ideally healthy or even optimally so: they are, however, as well cared for and as uniformly nourished as any group likely to be drawn from the extra-institutional population. That this economically homogeneous sampling of the general population presents smooth and regular curves of increment in stature and weight may be taken as evidence that use of the relatively small but biologically acceptable sampling offers satisfactory data for adequate reduction and therefore reliably illustrates the growth pattern.

THE USE OF AGE-WEIGHT-STATURE TABLES

Because of the common use of age-weight-stature tables it is profitable to observe the relatively low degree of weight-stature relationship. Reference to Figures 22 and 23 will show that investigators have obtained very different results when computing the weight-stature correlation. The divergence is particularly marked between the figures of Boas and our own. Despite the relatively high correlations which sporadically appear in practically all published studies it is our conviction that the weight-stature correlation is invariably too low for use in a sense as definitely predictive as that required for an age-weight-stature table.

The problem of the age-weight-stature table may be examined from another angle. In a group of children of identical age and identical weight the stature may range from one extreme of the distribution to the other. For example: of sixteen three month old girls each weighing $13\frac{1}{2}$ pounds the horizontal lengths range from 578 to 617 mm., falling indeed into all four quarters of the horizontal length distribution. It is nevertheless true that with an increase in average stature there is likewise an increase in average weight. The median weight of the girls in the four horizontal length quarters shows a progressive increase. The median weight of the girls in the lowest quarters of horizontal length is 11.75 pounds; in the second quarter 12.5 pounds; in the third quarter 13 pounds

and in the highest quarter 14 pounds. The range in weight of our three month old girls is 10 to 18 pounds.

The statures of thirteen three year old girls, each of whom weighs 32 pounds, range from 923 to 988 mm.; this 65 millimeter range covers all four quarters of the three year female stature distribution. The median weight of the girls in the lowest quarter is 29 pounds; in the second quarter 31.5 pounds; in the third quarter 33 pounds and in the highest quarter 36 pounds. The range in weight of three year old girls is 24 to 46 pounds.

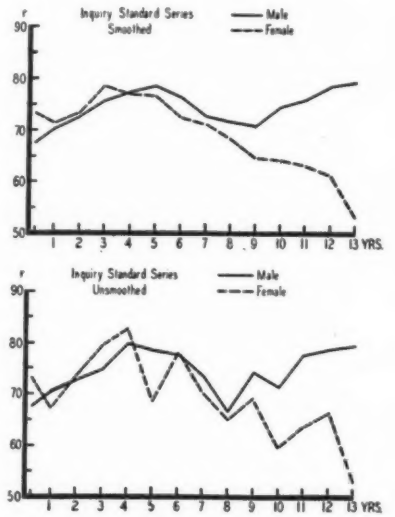

FIG. 22. Comparison of weight-stature correlation coefficients, male and female, Inquiry standards.

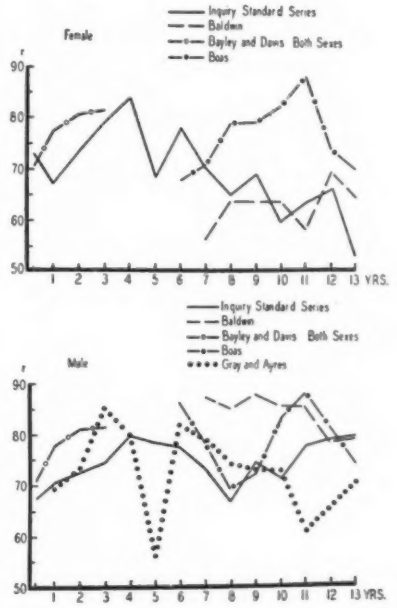

FIG. 23. Comparison of weight-stature correlation coefficients, male and female, from published samplings.

Statures of twelve five year old girls, each weighing 42 pounds, range from 1069 to 1157 mm., falling into all four quarters of the five year female stature distribution. The median weight of the lowest quarter is 37 pounds; of the second quarter 40 pounds; of the third quarter 44 pounds and of the highest quarter 47 pounds. The range in weight of five year old girls is 30 to 63 pounds.

As a last example, the statures of seven eleven year old girls, each weighing 88 pounds, range from 1404 to 1584 mm. The median weight of the lowest quarter of the eleven year female stature distribution is 77 pounds; of the second quarter 79 pounds; of the third quarter 85 pounds and of the highest quarter 99 pounds. Again, the stature range of 80 mm. for eighty eight pound girls covers

all four quarters of the eleven year female stature distribution. The range in weight of our eleven year old girls is from 62 to 120 pounds.

The contention set forth in the above paragraphs holds for both sexes and for all ages between three months and thirteen years as can be plainly seen in Table XIX. While we may reasonably conclude that weight and stature of chil-

TABLE XVIII. RANGES IN STATURE AND IN WEIGHT AND MEDIAN WEIGHT BY QUARTERS OF STATURE DISTRIBUTIONS, MALE AND FEMALE

Age	Range mm.	Male Weight Range lbs.	Median Weight lbs.	Range mm.	Female Weight Range lbs.	Median Weight lbs.
3 mo.—1st quarter—Horiz. Length	526– 600	10 – 15	13	536– 580	10 – 13½	11.75
2nd " " "	601– 610	12 – 16	14	581– 590	10½– 15	12.5
3rd " " "	611– 625	12 – 16	14	591– 605	11½– 15	13
4th " " "	626– 675	14 – 18½	15	606– 660	11½– 18	14
12 mo.—1st quarter—Horiz. Length	686– 745	18 – 24½	21.5	681– 725	16 – 22	19
2nd " " "	746– 760	18½– 26½	23	726– 740	17 – 26½	21
3rd " " "	761– 775	20 – 26½	23.75	741– 760	19 – 28½	22
4th " " "	776– 860	21½– 32½	25.5	761– 850	20½– 32½	23.5
2 yrs.—1st quarter—Stature	801– 855	22½– 30½	27	766– 845	21 – 28	24.75
2nd " "	856– 875	24 – 32	27.25	846– 860	23½– 31	27
3rd " "	876– 895	25½– 34½	29	861– 880	23 – 33½	27.5
4th " "	896– 980	26½– 42	30.5	881– 950	25½– 38	30
3 yrs.—1st quarter—Stature	861– 945	26 – 34½	30.5	861– 935	24 – 32	29
2nd " "	946– 960	27 – 37	32	936– 955	26 – 35	31.5
3rd " "	961– 990	28 – 39	33.75	956– 980	29 – 38½	33
4th " "	991–1075	31½– 46	36.25	981–1055	31 – 46	36
4 yrs.—1st quarter—Stature	936–1015	29 – 40½	35	941–1010	26 – 37	32.5
2nd " "	1016–1040	32½– 40½	35.5	1011–1030	30½– 40	36.25
3rd " "	1041–1065	34½– 50	39	1031–1060	31½– 44	37.5
4th " "	1066–1150	32 – 57	42	1061–1130	38 – 55½	41.5
5 yrs.—1st quarter—Stature	1001–1085	33 – 48	38	996–1075	30 – 46	37
2nd " "	1086–1105	36 – 47	41	1076–1100	33 – 71	40
3rd " "	1106–1140	39 – 52	45	1101–1130	35 – 52	44
4th " "	1141–1230	39 – 67	48	1131–1245	39 – 63	47
6 yrs.—1st quarter—Stature	1076–1140	38 – 51	42.5	1056–1145	36 – 51	43
2nd " "	1141–1165	40 – 53	46	1146–1175	38 – 55	46
3rd " "	1166–1205	43 – 62	50	1176–1205	42 – 72	49.5
4th " "	1206–1280	45 – 68	53	1206–1295	46 – 70	55
7 yrs.—1st quarter—Stature	1106–1205	42 – 55	48	1101–1205	40 – 58	47
2nd " "	1206–1235	45 – 62	53.5	1206–1235	44 – 62	51
3rd " "	1236–1275	49 – 67	55	1236–1260	45 – 69	56.5
4th " "	1276–1375	52 – 82	61	1261–1365	49 – 79	60
8 yrs.—1st quarter—Stature	1111–1270	38 – 66	55	1136–1260	47 – 63	54
2nd " "	1271–1300	50 – 71	60.5	1261–1290	46 – 68	57
3rd " "	1301–1340	55 – 91	61	1291–1330	58 – 91	64.5
4th " "	1341–1395	61 –105	67.5	1331–1420	57 – 84	67
9 yrs.—1st quarter—Stature	1236–1325	51 – 70	60	1266–1325	52 – 70	62
2nd " "	1326–1355	63 – 83	69	1326–1355	50 – 85	63.5
3rd " "	1356–1400	61 – 80	67	1356–1390	66 –106	74
4th " "	1401–1470	67 –106	77.5	1391–1485	63 –104	76
10 yrs.—1st quarter—Stature	1250–1379	56 – 76	66	1290–1369	56 – 97	66
2nd " "	1380–1419	60 – 92	78	1370–1419	61– 104	73.5
3rd " "	1420–1459	66 – 99	76	1420–1459	64 – 97	77
4th " "	1460–1529	60 –122	89	1460–1529	67 –113	85.5
11 yrs.—1st quarter—Stature	1300–1429	54 – 90	71	1350–1429	62 – 98	77
2nd " "	1430–1459	68 –108	79	1430–1479	64 –116	79
3rd " "	1460–1519	72 –104	83	1480–1519	62 –116	85
4th " "	1520–1659	78 –132	101	1520–1649	74 –120	99
12 yrs.—1st quarter—Stature	1320–1469	60 –110	75	1340–1519	63 –114	87
2nd " "	1470–1509	72 –120	83	1520–1559	84 –136	93
3rd " "	1510–1559	84 –124	95	1560–1599	84– 136	106
4th " "	1560–1729	80 –164	105	1600–1699	92 –138	117
13 yrs.—1st quarter—Stature	1370–1519	66 –110	85	1480–1559	90 –120	102
2nd " "	1520–1569	78 –122	97	1560–1589	96 –128	111
3rd " "	1570–1619	92 –132	106	1590–1639	102 –156	112
4th " "	1620–1819	92 –176	110	1640–1719	102 –156	118

dren show a real relationship, other factors operate to condition independently either of the measures. Therefore neither can be utilized as a criterion of the other at any age level. The need for separate standards of age-stature and age-weight is evident. From experience which does not fall within the limits of this discussion there is need for further investigation of the determiners of weight before even the age-weight standards can be used with confidence in assessment of the individual. Although a child may be placed above or below the mean of his group in weight, the importance of that deviation in appraisement of the child's physical status must be determined upon the basis of other factors not included in this analysis but glanced at in an earlier paragraph dealing with hydration of tissues.

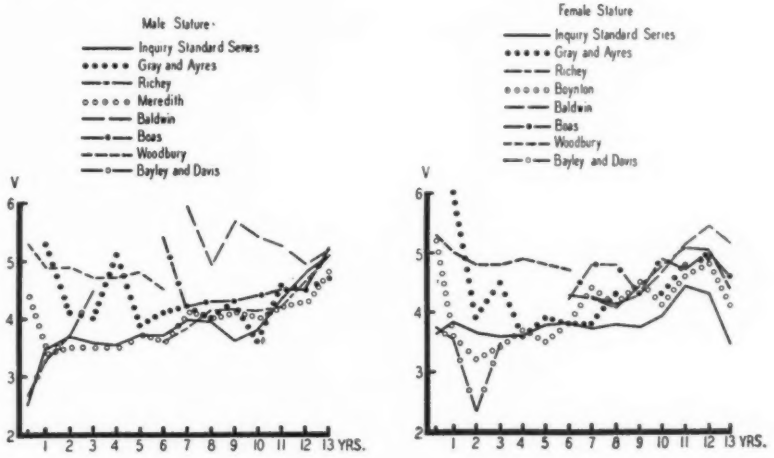

FIG. 24. Comparison of coefficients of variability in stature, male and female, from published samplings.

INACCEPTABILITY OF WEIGHT AS A CRITERION OF GROWTH

Investigations upon the growth of normal children lead to the conclusion that quantitatively there are many patterns of increment. The competence of the instrument to be used in the reduction of these patterns must be determined by both biological and statistical criteria following definition of the growth concept itself. When we measure the increment which accompanies maturation, or, as it is popularly known, "growing-up," we confine the concept to cellular multiplication (growth) within definite, meaningful limits. When we speak of age-typical size we imply an absence or an infrequency of extreme deviations from this size at the age in question. Indeed, when we measure a child, we premise a "size" progression which continues from conception to adulthood with variation but without recession. Nevertheless a child may lose weight though we do not say he shrinks. Weight indeed is a complex produced by many different deter-

miners which are not all concerned with growth. This view is substantiated by our chosen statistical constant of dispersion, namely the coefficient of variability (Figures 24 and 25).

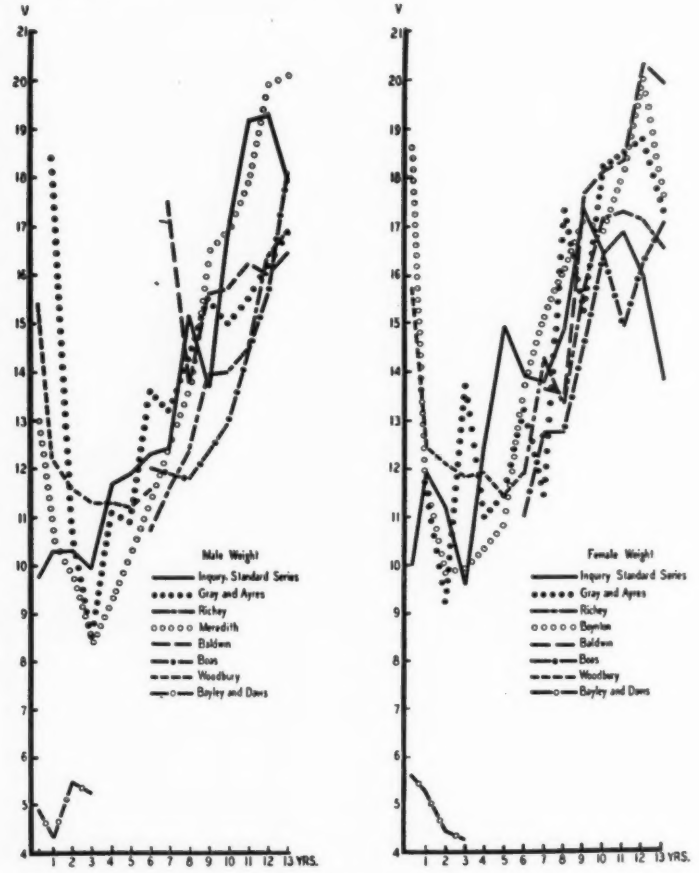

FIG. 25. Comparison of coefficients of variability in weight, male and female, from published samplings.

The greater homogeneity and consistency of the length measurements in comparison with weight are well known; nevertheless, weight has been used as a measure of growth so extensively that the superiority of stature seems worth emphasizing. Weight cannot be either as reliable or as valid an indicator of growth until it becomes possible to distinguish between weight which is a reflection of growth and weight which is really due to an increase in subcutaneous fat or to a fluctuation in tissue hydration.

SUMMARY

1) A long term study of the growth and development of healthy boys and girls of economically secure families has been in continuous progress at the Developmental Health Inquiry since 1931.
2) This report covers a modified longitudinal study of stature and weight measurements made on 493 boys and 459 girls of the Inquiry Standard Series. This group of well children, ranging from three months to thirteen years of age, is augmented by other selected groups. Children of the Standard Series are measured within a week of their birthdate.
3) Data are reduced by use of the punched card method with International Business Machines Equipment.
4) The Inquiry Standard Series is shown to be a homogeneous group in the two measures, stature and weight. No significant difference between the means occurs with a forty nine percent increase in the number of examinations made on children of identical age.
5) After two years of age stature is demonstrated to be a very competent instrument of prediction over annual intervals, weight somewhat less so. The tallest boys and girls and the shortest boys and girls consistently maintain their rank as well as those in the middle of the total stature distribution.
6) The constants presented as age standards are shown to be significantly different from one another. The overlapping of the distributions at successive examination ages is sufficiently small to validate the use of age means as age-typical standards.
7) Sex differences in mean stature are found to be significant at three, six, nine, twelve, eighteen months and two, two and one half, twelve and thirteen years. Sex differences in mean weight are found to be significant at three, six, nine, twelve, eighteen months and two, two and one half, three, twelve and thirteen years.
8) Annual increments in stature and in weight of boys and girls of the Inquiry Standard Series are presented. Mean increment in female stature decreases greatly during the first postnatal year, much less during the second year and continues to decrease until the eleventh year when there is a rise in increment. Mean increment in male stature follows the same pattern of velocity until the twelfth year when there is a rise in increment. Mean increment in weight of boys and girls during the first year of life is approximately double the birth weight. During the second and third years the weight increment decreases and beginning with the fourth year there is a rise in annual weight increment which is maintained through the twelfth year.
9) The stature-weight relationship is shown to be too low for employment of either measure as a criterion of the other. The use of the age-weight-stature table in individual appraisal is therefore discouraged.
10) As a measure of growth, stature is superior to weight.

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GROWTH RATIO OF THE FOWL'S TARSOMETATARSUS

I. VARIATION WITH RESPECT TO BODY WEIGHT

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Richards (1935) has presented a critical analysis of the constant differential growth ratio concept (Huxley 1932), wherein amongst other data he utilizes those of Moment (1935) on the growing rat. The α value (see Huxley and Teissier 1936 for the revised terminology of relative growth) for the eye is found by Richards to vary with the age of the population. The growth ratio for chela-weight in the pistol crab has also been found to vary with size (Dawes and Huxley 1934). The use of the concept of allometry in studies on the genetics of size and form depends to a great extent on the constancy of the differential growth ratios. If they vary through the growing period, then it is apparent for the purposes of determining genetic differences in growth patterns, should such exist, that single determinations of α from birth or hatching date to maturity will not suffice. If the variation of α is a function of some variable such as growth rate or body weight, growth ratios should be determined under comparable conditions of such independent variables. If on the other hand there is no regularity in the fluctuations of the growth ratio, its utility for such types of investigation is greatly impaired. In any case as has been pointed out by Huxley and by Davenport (1934) investigation of the behavior of α in individual organisms is of considerable importance.

Data for such study have been gathered in connection with a project of work on the growth of the domestic fowl, a report of the first stage of which has been published earlier (Lerner 1937). These data consist of measurements of shank lengths of chickens extending from 4 weeks to 20 weeks of age. The birds used were Barred Plymouth Rocks, S. C. White Leghorns and reciprocal first crosses as well as back-crosses to both parental breeds. Shank measurements were made at various ages using the measuring device described by Burmester and Lerner (1937). Since a large number of birds were killed before reaching the age of 20 weeks, the original population available for analysis was reduced to a total of 138 individuals, hatched in three lots. Body weights and shank measurements for all of these birds were available for the ages of 4, 8, 12, 16 and 20 weeks. A number of determinations at intermediate ages for some of the birds involved were not used for this analysis in order to present uniform data for all of the birds.

The 20 week shank measurement was available in the form of the length of the cleaned tarsometatarsus prepared in a manner described earlier (Lerner

1937). All of the live shank measurements were accordingly transformed into tarsometatarsal lengths by the use of the regression equation

(1) $$\text{tarsometatarsus} = 0.7863 \times \text{shank length} - 0.2481$$

derived from data on 475 birds dissected throughout the course of growth.

Individual determinations of the α values of the tarsometatarsal length with respect to body weight for each successive stage of growth were obtained by the formula

(2) $$\alpha = \frac{\log T_2 - \log T_1}{\log W_2 - \log W_1}$$

where T_2 and T_1 represent the lengths of the bone at the successive ages, corresponding to W_2 and W_1—body weights at the same respective ages. The values of α so obtained are probably not as exact as those obtained by a least squares fit, since the errors of measurement will greatly affect determinations obtained from formula (2). Yet with only two points available for each growth stage, this direct determination of the differential growth ratio serves the desired purpose. In such manner 552 different α values (four stages for each of the 138 individuals) were obtained and then subjected to an analysis of variance. It should be noted that since the dimensions on the two sides of the allometric equation are not equivalent, the tarsometatarsus yielding a linear measurement and body weight a mass one (see Lerner 1937), the α values here represent only one-third of the true growth ratio. The first step in the procedure was to determine if all of the various crosses belong to the same population with respect to their growth ratios. While Lerner (1937) has postulated that the growth pattern concerned is uniform throughout the normal-sized breeds of poultry, this determination was made on the total length of the growing period and not on the

TABLE 1. MEAN SQUARES BETWEEN AND WITHIN MEANS OF CROSSES

Sex	Male						Female					
Hatch	I		II		III		I		II		III	
Source of variance	Between crosses	Within crosses	Between crosses	Within crosses	Between crosses	Within crosses	Between crosses	Within crosses	Between crosses	Within crosses	Between crosses	Within crosses
Degrees of freedom	3	11	7	33	5	11	3	11	5	31	3	9
4– 8	.11	.13	.25	.15	.16	.29	.22	.06	.06	.08	.12	.17
8–12	.79	.36	.11	.24	.35	.67	.21	.22	.13	.31	.51	.91
12–16	1.73	.77	1.91	.75	.58	.87	1.84	1.64	1.14	.73	.71	2.97
16–20	.93	3.35	2.21	2.38	1.48	2.34	.32	1.65	2.31	2.31	.45	5.40
All ages	.89	1.15	1.12	.88	.64	1.04	.65	.89	.91	.86	.45	2.36

separate growth stages. However, examination of Table 1 shows that the variance within each cross ("cross" in this instance refers to purebreds, as well as F_1 hybrids and the backcrosses) for each age is not significantly less than the variance observed between crosses. As a matter of fact, it is greater within crosses in the majority of cases, both for the individual age groups and in case of the pooled variance for all ages, indicating lack of differences with respect to the α

values between the parental breeds and the various hybrids. This is a confirmation of the earlier hypothesis and permits considering together all individuals irrespective of descent.

The mean growth ratios for each sex, hatch and age are presented in Table 2, together with α values for the complete period studied (4 to 20 weeks of age) derived by substituting the two end ages instead of the successive ages in equation 2. The latter values indicate a significant sex difference, in favor of the males, similar to the one found in the material previously reported (Lerner 1937). An analysis of variance presented in Table 3 confirms this for the pooled variance, including all of the 552 separate determinations.

TABLE 2. MEAN GROWTH RATIOS OF THE TARSOMETATARSUS

Hatch	Sex	Number	Age period 4–8	8–12	12–16	16–20	4–20
I	male	15	.391	.509	.429	.421	.423
	female	15	.398	.485	.406	.379	.415
II	male	41	.381	.500	.454	.373	.425
	female	37	.389	.499	.417	.232	.405
III	male	17	.426	.519	.379	.544	.448
	female	13	.498	.552	.357	.488	.436
All hatches	male	73	.384	.506	.433	.427	.430± .0030
	female	65	.397	.506	.402	.317	.414± .0036

TABLE 3. MEAN SQUARES BETWEEN AND WITHIN SEXES

Source of variance	Degrees of freedom	Total squares	Mean square
Total	551	832.43	
Between sexes	1	17.25	17.25
Within sexes	550	815.18	1.48

Table 4 presents an analysis of variance of the α values with respect to differences prevailing between hatches. The mean squares obtained indicate that at least for some of the ages significant differences between hatches prevail. The significance of this finding will be discussed later, but it must be noted that for further analysis here the three hatches must be treated separately because of the differences in the mean growth ratios observed between them in some of the cases.

TABLE 4. MEAN SQUARES BETWEEN AND WITHIN HATCHES

Sex		Male		Female	
Source of variance		Between hatches	Within hatches	Between hatches	Within hatches
Degrees of freedom		2	70	2	62
Age	4– 8	1.42	.16	.50	.94
	8–12	.23	.35	1.81	.37
	12–16	3.79	.92	1.77	1.30
	16–20	22.90	2.38	35.45	2.45
	All ages	3.05	1.21	10.08	1.71

Table 5 presents the final breakdown of the total variance into variance arising from variation of different individuals of the same age (between individuals), variance due to variation of α values between ages of the same individual (between ages) and the interaction, serving here as the estimate of experimental error. The mean square between individuals is found in the great

TABLE 5. VARIANCE BETWEEN AGES AND INDIVIDUALS

Sex Hatch	Source of variance	Degrees of freedom	Male Total squares	Mean square	Degrees of freedom	Female Total squares	Mean square
I	Between ages	3	11.43	3.81	3	9.93	3.31
	Between individuals	14	8.37	.60	14	5.98	.43
	Interaction	42	53.04	1.26	42	41.04	.98
II	Between ages	3	40.80	13.60	3	138.90	46.30
	Between individuals	40	28.07	.70	36	35.14	.98
	Interaction	120	119.80	1.00	108	89.20	.83
III	Between ages	3	30.46	10.15	3	27.77	9.26
	Between individuals	16	6.14	.38	12	22.89	1.91
	Interaction	48	52.48	1.09	36	67.49	1.88
All hatches	Between ages	3	46.84	15.61	3	117.71	39.24
	Between individuals	72	48.68	.68	64	94.16	1.47
	Interaction	216	261.17	1.21	192	246.62	1.28

majority of cases to be less than that of the interaction. In no case is it significantly higher. On the other hand the variance between ages is without exception of great significance. This indicates that the α value does indeed vary with age in the same individual. It is important now to determine whether this effect is actually due to age or to some other factor which bears an intimate correlation with age, such as body weight or rate of growth. From the data on hand it is not possible to differentiate between the effects of age and of body weight on the fluctuations in the α values. This is because the variation in body weight here is largely discontinuous, each age having a practically non-overlapping range of weights, due to the fact that the observations were taken at set ages. There is a number of overlapping cases, but it is too small to separate age from weight effects. Thus when variance of all males is isolated within body weights, rather than within ages the estimate of experimental error yields a mean square of 1.12 as compared to 1.08, when the age classification is used. Consequently, age as used here for a criterion of classification may mean either chronological age or morphological age (body weight).

It is likely that the differences observed between hatches in Table 4 are related to the differences observed between ages. It has been demonstrated by numerous investigators (see Lerner and Asmundson 1938) that date of hatch has an effect on the rate of growth and hence on the body weight at different ages. It does not appear probable that the variation in α is a simple consequence of diminishing growth rate as it is in the crusher of the pistol crab (Dawes and Huxley 1934), or as a consequence of the fact that the growth curve of the parts as well as of the whole is sigmoid (Lumer 1937). Neither does the curve of α parallel that of the growth rate as it seems to be in Moment's rapidly growing rats (Richards 1935). The relative growth rate as expressed by the differences

between the natural logarithms of body weight at any two successive ages is constantly diminishing in the case of the material here presented. With the exception noted below the α value on the other hand in general rises to 12 weeks and then decreases to 20 weeks. However, the individuals of the third hatch show a higher α value for the 16–20 week stage than for the 12–16 week stage. Coincidentally, there is in this hatch a large number of individuals which show an increase in rate of growth in the last stage. It is then likely that a relation between growth rate and the α value does exist, although it cannot be expressed as a simple proportionality. A somewhat similar situation seems to prevail in the slowly growing rats of Moment (Richards 1935).

It may seem that the trend in α values parallels the curve of absolute increments of body weight, especially since the denominator in formula (2) is a function of such increments. Examination of the data, however, shows that no relation exists between the two curves.

It thus appears that, for the material on hand, the differential growth ratio is not a constant but varies with age. When a log-log plot is made for all ages an apparently straight line results. Because of the large range of body weight, a good least squares fit to a straight line can be made, such as has been obtained on similar material reported on previously (Lerner 1937).

The demonstration here presented should not be confused with breaks observed in the log-log straight line relationships of part and whole in many forms (Huxley 1932). What is being dealt with here at least in part is the masking effect of mass observations on the true course of relative growth, such as has been claimed by Richards (1935),[1] the variation found within individuals compensating each other to a certain extent. However, mass law should not be invoked here to explain this situation (as it has been done in a general form by Teissier 1936), since it is obvious that even the average trends (Table 2) present variations in a uniform direction. Actual changes in values are found to exist here.

In view of this finding, it is necessary to conclude that for any purposes of comparison of individuals or groups of individuals with respect to their differential growth ratio, no single determination of α is sufficient. A number of these must be made for different ages, body weights or growth rates depending on the causality of the fluctuation of α which still remains to be determined.

Clerical assistance under the auspices of WPA is acknowledged.

SUMMARY

The differential growth ratio α for the growth of the tarsometatarsus with respect to body weight in the domestic fowl is found to fluctuate in different stages of growth. While appearing as a constant for the whole period from 4 to 20 weeks of age, in reality, as observed in individual birds, it tends to increase from 4 to 12 weeks and to decrease from 12 to 20 weeks of age.

[1] Since this was written A. H. Hersh (*Science* **87**: 237, 1938) has published a note drawing attention to the difference between average and individual α values in the growth of the legs of *Notonecta undulata*, the common backswimmer.

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GROWTH RATIO OF THE FOWL'S TARSOMETATARSUS

II. VARIATION WITH RESPECT TO TIME

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The relative growth function of Huxley (Huxley and Teissier 1936)

$$y = bx^{\alpha} \tag{1}$$

can be derived on the assumption that the growth of the whole x and of the part y behave as parabolic functions of time. Thus if

$$x = b_1 t^{\beta_1} \tag{2}$$

and

$$y = b_2 t^{\beta_2} \tag{3}$$

it can be readily seen that α is the quotient of β_2 over β_1.

The determination of the exponent β_1 from two successive points on a growth curve can be made from the equation

$$\beta_1 = \frac{\log x_2 - \log x_1}{\log t_2 - \log t_1}. \tag{4}$$

Similarly β_2 and α can be determined from corresponding equations. Ludwig (1929) has presented equation (4) in lieu of Schmalhausen's (1927) growth quotient formula by way of determination of the specific rate of growth.

Lumer (1937) has pointed out in this connection that growth over extended periods of time is sigmoid in nature and not parabolic. Furthermore, he showed that when α is derived on the assumption that x and y are simple autocatalytic functions of time, α depends not only on the growth quotients β_1 and β_2 but also on the upper asymptotes of the curves of x and of y. However, for the early stages of growth this is less applicable, since here it is difficult to distinguish the simple autocatalytic function from an exponential one. Thus, when the parts of the x and y curves which approach the final values of x and y are not considered, short segments of the respective growth curves may be expressed in the form of equations (2) and (3). Such a procedure while not shedding any light on the absolute nature of α may produce some evidence for the causes of its fluctuations as observed in the comparatively early stages of growth of the fowl by Lerner (1938). Teissier (1936) has also derived the relative growth ratio as a quotient of parameters of food supply and utilization for the part and for the whole. These constants, however, are not available and hence cannot be used for the analysis proposed.

The writer has shown that α for the length of the tarsometatarsus with respect to body weight is not a constant for the four four-weekly stages of growth from 4 to 20 weeks of age. If α is considered as the quotient of β_2 over β_1 then it follows that the variations of the latter two from stage to stage in this period cannot follow identical trends, since under such conditions α would remain a constant for the whole of the period studied.

Hence β_1 and β_2 were determined separately for the same population of 138 individuals and an analysis of their variation is here presented. It should be noted that β_1 and β_2 are expressed in different dimensions. While the former deals with a mass, the latter deals with a linear measurement, hence necessitating that β_2 should be corrected by the factor 3.

Although the same population was used in the earlier report, the values therein presented differ somewhat from the quotients which may be obtained here by dividing β_1 into β_2. This follows from the fact that the mean values previously reported were obtained by averaging the α's themselves and hence were unweighted for the differences between the β's for the different individuals. Here, on the other hand, the β's themselves were averaged so that mean α's may be considered as having been weighted. This difference in procedure, while modifying somewhat the absolute α values, does not change the character or direction of the trends observed.

The 138 individuals measured arose from three different hatches. Since as shown in the previous paper α did not differ in birds from different ancestry, the B. P. Rocks, S. C. W. Leghorns and Rock×Leghorn crosses and backcrosses represented in this population were here treated together. The mean growth quotients for the tarsometatarsus (β_2) and for body weight (β_1) with their standard errors for each hatch, sex and stage of growth are presented in Table 1. Beginning of incubation rather than time of hatch was considered in these calculations as the age origin (Schmalhausen 1928). This is a somewhat arbitrary procedure, since the age of the whole body is greater than that of the limb. Hence an error of, perhaps, one half of a week is being introduced here.

TABLE 1. WEIGHTED MEAN GROWTH QUOTIENTS FOR TARSOMETATARSUS AND BODY WEIGHT

Sex	Hatch	Tarsometatarsus				Body Weight			
		4–8	8–12	12–16	16–20	4–8	8–12	12–16	16–20
Male	I	1.22±.039	1.02±.028	.75±.058	.61±.102	3.14±.089	2.03±.072	1.73±.082	1.55±.107
	II	1.17±.023	1.05±.023	.72±.027	.56±.038	3.02±.052	2.10±.045	1.59±.086	1.46±.063
	III	1.10±.040	.89±.037	.67±.034	.73±.052	2.60±.086	1.73±.081	1.77±.073	1.41±.151
Female	I	1.15±.043	.92±.066	.63±.058	.52±.032	2.88±.088	1.92±.077	1.57±.078	1.40±.072
	II	1.14±.022	1.02±.028	.59±.032	.29±.028	2.93±.047	2.05±.051	1.42±.051	1.26±.060
	III	1.11±.053	.84±.043	.50±.056	.48±.025	2.64±.095	1.58±.117	1.55±.173	1.10±.110

Table 2 presents the analysis of variance and covariance of the growth quotient values appearing in Table 1. Covariance due to the low number of degrees of freedom for the various subclasses is of limited value in interpreting the relationship of the two quotients. The regression unit for tarsometatarsus cor-

responding to isometry is 0.333 rather than unity because of the difference in the dimensions of the two variables. Thus, the regression between sexes and ages shows positive, while that between hatches shows negative allometry.

TABLE 2. VARIANCE AND COVARIANCE OF MEAN GROWTH QUOTIENTS

Source of variance	Degrees of freedom	Tarsometatarsus Sum of squares	Tarsometatarsus Mean square	F	Body Weight Sum of squares	Body Weight Mean square	F	F at P = .05	F at P = .01	Sum of products	r	r at P = .05	r at P = .01	Regression
Total	23	1.6396			8.4928					3.3780	.930	.404	.515	.398
sexes	1	.0704	.0704	33.52	.1397	.1397	26.87	5.99	13.74	.0991	—	—	—	.709
ages	3	1.4450	.4817	229.38	7.8476	2.6159	503.06	4.76	9.78	3.1501	.935	.950	.990	.401
hatches	2	.0157	.0079	3.76	.2350	.1175	22.60	5.14	10.92	.0562	.926	.997	1.000	.239
Interactions														
sex-age	3	.0277	.0092	4.38	.0153	.0051	†	4.76	9.78	.0205	.995	.950	.990	1.340
sex-hatch	2	.0006	.0003	†	.0018	.0009	†	5.14	10.92	−.0007	−.700	.997	1.000	−.389
age-hatch	6	.0674	.0112	5.33	.2223	.0371	7.13	4.28	8.47	.0392	.320	.707	.834	.176
Exp. Error	6	.0128	.0021		.0311	.0052				.0136	.680	.707	.834	.437

† Variance less than experimental error.

The latter may be due to the dissimilarity observed in the behavior of the two quotients with respect to differences between hatches. While the F value for the body weight quotient shows significant variance as far as this criterion of classification is concerned, the corresponding F for the tarsometatarsal quotient falls considerably below the minimum level of significance.

The reason for this seems somewhat obscure. The difference between hatches in the body weight quotients is to be expected, since early growth is inhibited in later hatches and compensatory growth, should such exist in this case, does not evidence itself till after 20 weeks of age, as shown by Lerner and Asmundson (1938). On the other hand, while there is some depression of the tarsometatarsal growth quotient in the later hatches it is apparently of no statistical significance. The variation of α due to time of hatch reported for this population earlier is then a consequence of the differences in rate of growth of the whole body and may be independent of the growth quotient for the tarsometatarsus.

A clearer picture of the covariation of the body weight and the tarsometatarsal growth quotients may be obtained if these quotients are expressed in relative terms. Thus, if for each hatch-sex group each of the two quotients for the first stage observed (4–8 weeks) is taken as a base of 100, the remaining quotients may be expressed in relation to it as percentages.

Table 3 presents such relative values. A striking difference in the nature of variation between the quotients for body weight and the tarsometatarsus may be noted. While the growth quotient for the latter in the second stage is in the neighborhood of 80 percent of that of the first, the quotient for body weight exhibits a much bigger decrease in this stage, amounting to less than 70 percent on the average. In the next stage it is the tarsometatarsal quotient that shows a relatively greater drop as is also true of the fourth stage. This, of course, agrees with the trend of fluctuations of α.

It thus seems that the growth pattern of the tarsometatarsus is different from that of the body weight. The successive growth quotients of the former if

TABLE 3. GROWTH QUOTIENTS EXPRESSED AS PERCENT OF QUOTIENT FOR THE 4–8 WEEK STAGE

Sex	Hatch	Tarsometatarsus				Body Weight			
		4–8	8–12	12–16	16–20	4–8	8–12	12–16	16–20
Male	I	100	84	61	50	100	65	55	49
	II	100	90	62	48	100	70	53	48
	III	100	81	61	66	100	67	68	54
Female	I	100	80	55	45	100	67	55	49
	II	100	89	52	25	100	70	48	43
	III	100	76	45	43	100	60	59	42

plotted with age as the abcissa would give a convex curve; that of the latter a concave one. If the true age of the tarsometatarsus could be determined a somewhat different picture may prevail, but the error of half a week can hardly change the trend to any great extent.

It is obviously not profitable to generalize from the case here presented and explain other situations, where α may be shown not to behave as a constant in a similar manner. However, it seems from this material that a closer inspection of the components of α may be of considerable value.

Assistance in computations under auspices of the WPA is acknowledged.

SUMMARY

The fluctuations in the value of the relative growth ratio of the tarsometatarsus with respect to body weight previously reported are shown to arise as a result of dissimilarity of the growth patterns of the component variables entering into its determination.

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IDENTIFICATION OF NORMAL STAGES IN CHICK EMBRYOS

By

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In order to have an easy and reliable means of testing the normality of chick embryos used in experimental work, the following relation between weight and crown-rump length has been employed:

$$I = \frac{1000 \times \sqrt[3]{W}}{L}.$$

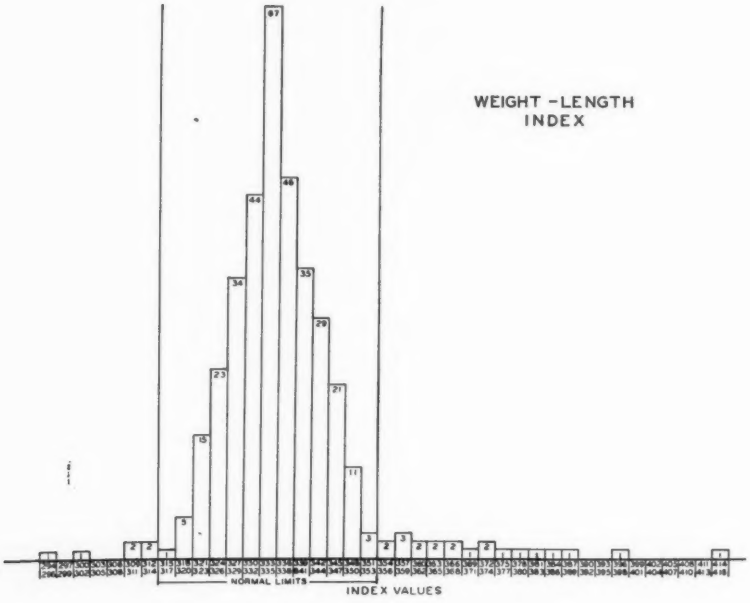
The factor of 1000 has been added merely to express the index values as whole numbers. Applied to a series of 361 chicks used in developmental studies, a reasonably good frequency curve has been set up with a mean represented by index values of 333 to 335 and extreme limits of 315 and 353. Chicks whose index values lay outside the above range were ruled out as abnormal. This index is readily calculable and involves no subjective methods.

In all experimental work, it is imperative that there should be some means for estimating the normality of the animals used. This is especially true in experimental embryology, where the precise stage of the embryo and its functional normality must be determined. For the chick, to be sure, there are such works as Keibel and Abraham's "Normaln-Tafeln" and Duval's "Atlas d'Embryologie," containing excellent plates showing body form and gross morphological relationships of embryos at various stages. These have been used extensively in the past as guides for the determination of normal development. Such methods of estimation, however, are not entirely satisfactory in view of the difficulty in the interpretation of stages, especially in the latter part of the incubation time. Romanoff and Grover ('36) have regularly candled eggs prior to use in various types of experiments, and there, likewise, the exact stage and the normality of the embryo is difficult of interpretation. Incubation age itself, although commonly used, is unreliable as a criterion of the extent of development, owing to the relatively large variations which may exist between embryos which have been incubated for the same period of time. Schmalhausen ('26) bases the normality of the embryos used in his experiments on the values obtained from the cube root of the weight, which can be easily and accurately measured. However, in view of the fact that functional disturbances during development resulting from maladjustments such as are not infrequently associated with hormonal unbalance may affect either weight or length independently, a relationship between these two quantities might be expected to yield a greater certainty of normality of proportion and hence of function.

A relationship of this sort which has often been employed is that of some definite linear measurement, such as crown-rump length, and the cube root of the weight. Such a relationship should be constant throughout the course of development. This was found to be true in the case of the so-called "pelidisi index" employed by von Pirquet ('16), as Stevenson ('34) pointed out. He and his co-workers find that it remains constant throughout the entire span of

life, and in man, even slight racial differences have been brought out by the use of this index value. He also finds that a deviation of more than three times the standard deviation from the norm indicates a variation greater than can be ascribed to mere chance, showing an individual abnormal in respect to nutrition or some other factor.

It would appear that a similar index might be very valuable if applied to chick embryos used in experimental work in the hope of deciding the important question of functional normality. The value obtained by dividing the cube root



of the wet weight of the chick (in grams) by the crown-rump length (in cms.) has been multiplied by a factor of 1000 merely to express the quotient as a whole number rather than as a fraction.

$$I = \frac{1000\sqrt[3]{W}}{L}.$$

Using this formula, index values were calculated for each of a group of 361 chicks used in 3 series of experiments dealing with measurements of changes in certain physical properties of the extra-embryonic fluids. The embryos ranged in age from about the 7th to about the 19th days of incubation. All eggs used were supplied by the same farmer and came from a large flock of Barred Plymouth Rock pullets. Incubation was carried out in a regulation Thelco incubator, set to operate at 38.5° C.

The histiograms obtained for each series of embryos have been combined into a single chart in the accompanying figure. It is at once apparent that a reasonably good frequency curve could be constructed from this histiogram, with a mean represented by the class 333–335 and its extreme limits by the classes 315–317 and 351–353. It should be pointed out here that these limits do not coincide with the range of normal variability (three times the standard deviation) mentioned in connection with Stevenson's work. However, nearly all measurements made on the fluids of those chicks whose index value lay outside the range stated above showed pronounced deviations in comparison with the figures for chicks within the range. The general features of the curve also indicate that these limits are reasonable. Accordingly, chick embryos with an index value between 315 and 353 have been considered as "normal" individuals, while embryos whose indices did not fall within these limits have been classed as "abnormal."

As far as the physical measurements (specific conductance, surface tension and hydrogen-ion concentration) are concerned, however, the fact that the deviations of these "abnormal" individuals are of either a "plusward" or a "minusward" nature is apparently without significance. Furthermore, inspection of the data discloses that sex does not modify the values of the index in any respect.

SUMMARY

Based on two readily determinable quantities, wet weight and crown-rump length, an index is proposed as an aid in the determination of the functional normality of chick embryos used in experimental work. It is believed that its application will enable the investigator to rule out various non-typical cases which might otherwise be included in the experimental data.

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STUDIES ON DENTAL CARIES
III. A METHOD OF DETERMINING POST-ERUPTIVE TOOTH AGE[1]

By

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INTRODUCTION

The concept, that attack by disease is often related to duration of exposure to a causative agent, is amply supported by an array of epidemiological studies. The application of this knowledge in the problem of dental caries leads to a recognition of the fact that attack of the teeth by caries may be, in part, a function of the length of time individual teeth are exposed in the mouth after eruption.[2] Support for this view may be derived from the well established fact that dental caries progressively increases with advancing chronological age.

Several characteristics of the development of the human dentition may be considered in the light of these preliminary remarks. It is well known that the several permanent teeth erupt according to definite sequential patterns and that as many as 12 to 20 years may separate the time of eruption of the first and the last permanent tooth. During this interval, the full complement of teeth in the permanent dentition are erupted into the oral cavity. It follows, therefore, that each of the separate corresponding pairs of permanent teeth,[3] at any designated chronological age, has been exposed in the mouth for different lengths of time.

On the basis of these considerations, a study of the duration of post-eruptive mouth exposure of each of the separate permanent teeth becomes of some interest in a study of the constitution of the caries problem. Since it may be postulated that attack by caries may be related to duration of exposure of the teeth in the mouth, it becomes of some practical importance to consider methods by means of which post-eruptive mouth ages of the separate permanent teeth may be quantitatively measured and compared.

METHOD

A direct approach to the measurement of duration of exposure of teeth in the mouth, post-eruptive tooth age, would appear to require repeated or continued observations of the same children during the development of their dentitions. Such observations, first recorded at the time each tooth erupts into the mouth, would permit the calculation of the actual post-eruptive age of each tooth at any point of time on the scale of chronological age. Clearly, the col-

[1] From the Division of Public Health Methods, National Institute of Health, U. S. Public Health Service.

[2] It is generally agreed that actual carious lesions appear in the teeth only after they have erupted into the oral cavity.

[3] Thus far it has not been shown that there are significent differences between the time of eruption of corresponding teeth (right or left). Cattell (1) and Klein, Palmer and Kramer (2).

APPENDIX TABLE

Values of $\int_{-\infty}^{X}\left[\int_{-\infty}^{X}\frac{1}{\sqrt{2\pi}}e^{-(1/2)X^2}dX\right]dX$ for Values of X from -4.0 to $+4.0$

X	Area under the integral of the normal curve from $-\infty$ to X	X	Area under the integral of the normal curve from $-\infty$ to X
−4.0	.00001	0.0	.39894
−3.9	.00001	.1	.45093
−3.8	.00002	.2	.50689
−3.7	.00003	.3	.56675
−3.6	.00004	.4	.63043
−3.5	.00006	.5	.69779
−3.4	.00009	.6	.76867
−3.3	.00013	.7	.84287
−3.2	.00018	.8	.92020
−3.1	.00027	.9	1.00043
−3.0	.00038	1.0	1.08331
−2.9	.00054	1.1	1.16862
−2.8	.00076	1.2	1.25610
−2.7	.00106	1.3	1.34552
−2.6	.00146	1.4	1.43667
−2.5	.00200	1.5	1.52930
−2.4	.00272	1.6	1.62324
−2.3	.00366	1.7	1.71829
−2.2	.00489	1.8	1.81427
−2.1	.00647	1.9	1.91105
−2.0	.00849	2.0	2.00849
−1.9	.01105	2.1	2.10647
−1.8	.01427	2.2	2.20489
−1.7	.01829	2.3	2.30366
−1.6	.02324	2.4	2.40272
−1.5	.02930	2.5	2.50200
−1.4	.03667	2.6	2.60146
−1.3	.04552	2.7	2.70106
−1.2	.05610	2.8	2.80076
−1.1	.06862	2.9	2.90054
−1.0	.08331	3.0	3.00038
− .9	.10043	3.1	3.10027
− .8	.12020	3.2	3.20018
− .7	.14287	3.3	3.30013
− .6	.16867	3.4	3.40009
− .5	.19779	3.5	3.50006
− .4	.23043	3.6	3.60004
− .3	.26675	3.7	3.70003
− .2	.30689	3.8	3.80002
− .1	.35093	3.9	3.90001
−0.0	.39894	4.0	4.00001

lection of such seriatim observations would involve a laborious field study extending over a considerable period of time. Satisfactory observations of this character, obtained in sufficient numbers, and including sufficient collateral data on the date of attack of the individual teeth by caries, are not readily obtained. In the light of this situation it becomes desirable to consider possible indirect methods for the determination of tooth age.

A specific approach to the problem of the measurement of duration of exposure of teeth in the mouth may be based on an extension of a basic concept given in a previous paper (2) in this series of publications. The analysis presented in that paper indicated that a reasonably satisfactory mathematical description of the age distribution of eruption of the separate morphological types of permanent teeth can be obtained by means of the normal probability (Gaussian) frequency function fitted to observations on the proportion of children having a given permanent tooth erupted in the mouth at successive chronological ages. The study indicates that the percentages of children at successive ages who had a given permanent tooth present in the mouth would, when plotted on arithmetic probability paper, result in a sufficiently regular linear progression to permit the determination of the constants (the mean and standard deviation) of a normal probability function representing the age distribution of eruption

TABLE 1

MEANS AND STANDARD DEVIATIONS OF NORMAL PROBABILITY CURVES DESCRIBING THE AGE DISTRIBUTION OF THE ERUPTION OF PERMANENT TEETH. 4,416 BOYS AND GIRLS, HAGERSTOWN, MARYLAND

Teeth	Upper Jaw (Corresponding Teeth)				Lower Jaw (Corresponding Teeth)			
	Mean (years)		Stand. Dev. (years)		Mean (years)		Stand. Dev. (years)	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Central Incisors	7.49	7.20	0.75	0.75	6.50	6.19	0.74	0.70
Lateral Incisors	8.62	8.15	1.10	0.94	7.64	7.31	0.80	0.75
Canine	11.80	11.05	1.42	1.37	10.70	9.85	1.15	1.15
First Premolar	10.42	10.00	1.50	1.40	10.75	10.20	1.42	1.41
Second Premolar	11.18	10.82	1.68	1.60	11.45	11.00	1.77	1.70
First Molar	6.64	6.54	0.75	0.70	6.44	6.12	0.75	0.87
Second Molar	12.70	12.40	1.18	1.40	12.20	11.90	1.20	1.55

of that tooth. Given the constants for the equation of the normal curve of eruption of any specified permanent tooth, it is possible to obtain estimates of a number of useful facts. From the equation for the normal curve itself, the instantaneous rate of eruption of a tooth *at* any age may be calculated. From the integral of the normal curve, the percentage of children having the tooth erupted *at* any age may be determined. The difference in the value of the integral of the normal curve, between two different ages, gives the percentage of children having the tooth erupt *during* the interval between the two ages. It is also possible, since chronological age is expressed in terms of time (years), to use the double integral of the normal curve as a mathematical expression for the accumulated post-eruptive tooth years of exposure for any tooth. Thus, if x represents age in years, m_x and σ_x the mean and standard deviation, respectively, of the normal curve describing the age distribution of eruption of that tooth, the accumulated years of exposure of the particular tooth up to any given age, x, may be represented by the function:

TABLE 2

PERCENT OF CHILDREN HAVING PERMANENT TOOTH ERUPTED INTO THE MOUTH AND POST-ERUPTIVE TOOTH AGE (ACCUMULATED TOOTH YEARS OF EXPOSURE). DATA FOR EACH TYPE OF CORRESPONDING PAIRS OF PERMANENT TEETH OF BOYS AND GIRLS FOR EACH CHRONOLOGICAL YEAR OF AGE FROM 4.5 TO 17.5. BASED ON ANALYSES OF DENTAL EXAMINATIONS OF 4,416 ELEMENTARY SCHOOL CHILDREN, HAGERSTOWN, MARYLAND

Chronological Age (years)	Central Incisors			Lateral Incisors			Canines			1st Pre Molars			2d Pre Molars			1st Molars			2nd Molars		
	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]
Boys, Upper Jaw																					
4.5	− 3.99	.00	.00	− 3.75	.01	.00	−5.14	.00	.00	−3.95	.00	.00	−3.98	.00	.00	− 2.85	.22	.00	−6.95	.00	.00
5.5	− 2.65	.40	.00	− 2.84	.23	.00	−4.44	.00	.00	−3.28	.05	.00	−3.38	.04	.00	− 1.52	6.43	.02	−6.10	.00	.00
6.5	− 1.32	9.34	.03	− 1.93	2.68	.01	−3.73	.01	.00	−2.61	.45	.00	−2.79	.26	.00	− .19	42.47	.23	−5.25	.00	.00
7.5	+ .01	50.40	.30	− 1.02	15.39	.09	−3.03	.12	.00	−1.95	2.56	.01	−2.19	1.43	.01	+ 1.15	87.49	.91	−4.41	.00	.00
8.5	+ 1.35	91.15	1.04	− .11	45.62	.38	−2.32	1.02	.00	−1.28	10.03	.07	−1.60	5.48	.04	+ 2.48	99.34	1.86	−3.56	.02	.00
9.5	+ 2.68	99.63	2.01	+ .80	78.81	1.01	−1.62	5.26	.03	− .61	27.09	.25	−1.00	15.87	.14	+ 3.81	99.99	2.86	−2.71	.34	.00
10.5	+ 4.01	100.00	3.01	+ 1.71	95.64	1.90	− .92	17.88	.14	+ .05	51.99	.64	− .40	34.46	.39	+ 5.15	100.00	3.86	−1.86	3.14	.01
11.5	+ 5.35	100.00	4.01	+ 2.62	99.56	2.88	− .21	41.68	.43	+ .72	76.42	1.29	+ .19	57.53	.84	+ 6.48	100.00	4.86	−1.02	15.39	.09
12.5	+ 6.68	100.00	5.01	+ 3.53	99.98	3.88	+ .49	68.79	.98	+1.39	91.77	2.14	+ .79	78.52	1.53	+ 7.81	100.00	5.86	− .17	43.25	.38
13.5	+ 8.01	100.00	6.01	+ 4.44	100.00	4.88	+1.20	88.49	1.78	+2.05	97.98	3.09	+1.38	91.62	2.38	+ 9.15	100.00	6.86	+ .68	75.17	.98
14.5	+ 9.35	100.00	7.01	+ 5.35	100.00	5.88	+1.90	97.13	2.71	+2.72	99.67	4.08	+1.98	97.61	3.34	+10.48	100.00	7.86	+1.53	93.70	1.84
15.5	+10.68	100.00	8.01	+ 6.25	100.00	6.88	+2.61	99.55	3.71	+3.39	99.97	5.08	+2.57	99.49	4.32	+11.81	100.00	8.86	+2.37	99.11	2.80
16.5	+12.01	100.00	9.01	+ 7.16	100.00	7.88	+3.31	99.95	4.70	+4.05	100.00	6.08	+3.17	99.92	5.33	+13.15	100.00	9.86	+3.22	99.94	3.80
17.5	+13.35	100.00	10.01	+ 8.07	100.00	8.88	+4.01	100.00	5.69	+4.72	100.00	7.08	+3.76	99.99	6.32	+14.48	100.00	10.86	+4.07	100.00	4.80
Boys, Lower Jaw																					
4.5	− 2.70	.35	.00	− 3.93	.00	.00	−5.54	.00	.00	−4.31	.00	.00	−3.92	.00	.00	− 2.59	.48	.00	−6.42	.00	.00
5.5	− 1.35	8.85	.03	− 2.68	.37	.00	−4.64	.00	.00	−3.62	.01	.00	−3.36	.04	.00	− 1.25	10.56	.04	−5.58	.00	.00
6.5	.00	50.00	.30	− 1.43	7.64	.03	−3.75	.01	.00	−2.93	.17	.00	−2.80	.26	.00	+ .08	53.19	.33	−4.75	.00	.00
7.5	+ 1.35	91.15	1.03	− .18	42.86	.25	−2.86	.21	.00	−2.24	1.25	.01	−2.24	1.25	.01	+ 1.41	92.07	1.08	−3.92	.00	.00
8.5	+ 2.70	99.65	2.00	+ 1.08	85.99	.92	−1.96	2.50	.01	−1.55	6.06	.04	−1.67	4.75	.04	+ 2.75	99.70	2.06	−3.08	.10	.00
9.5	+ 4.05	100.00	3.00	+ 2.33	99.01	1.87	−1.07	14.23	.08	− .86	19.49	.16	−1.11	13.35	.12	+ 4.08	100.00	3.06	−2.25	1.22	.01
10.5	+ 5.41	100.00	4.00	+ 3.58	99.98	2.86	− .18	42.86	.35	− .17	43.25	.46	− .55	29.12	.33	+ 5.41	100.00	4.06	−1.42	7.78	.04
11.5	+ 6.76	100.00	5.00	+ 4.83	100.00	3.86	+ .71	76.11	.95	+ .52	69.85	1.03	+ .01	50.40	.72	+ 6.75	100.00	5.06	− .58	28.10	.21
12.5	+ 8.11	100.00	6.00	+ 6.08	100.00	4.86	+1.61	94.63	1.83	+1.21	88.69	1.83	+ .57	71.57	1.33	+ 8.08	100.00	6.06	+ .25	59.87	.64
13.5	+ 9.46	100.00	7.00	+ 7.33	100.00	5.86	+2.50	99.38	2.80	+1.90	97.13	2.77	+1.13	87.08	2.13	+ 9.41	100.00	7.06	+1.08	85.99	1.38
14.5	+10.81	100.00	8.00	+ 8.58	100.00	6.86	+3.39	99.97	3.80	+2.59	99.52	3.76	+1.70	95.54	3.06	+10.75	100.00	8.06	+1.92	97.26	2.32
15.5	+12.16	100.00	9.00	+ 9.83	100.00	7.86	+4.29	100.00	4.80	+3.28	99.95	4.76	+2.26	98.81	4.03	+12.08	100.00	9.06	+2.75	99.70	3.30
16.5	+13.51	100.00	10.00	+11.08	100.00	8.86	+5.18	100.00	5.80	+3.97	100.00	5.76	+2.82	99.76	5.02	+13.41	100.00	10.06	+3.58	99.98	4.30
17.5	+14.86	100.00	11.00	+12.33	100.00	9.86	+6.07	100.00	6.80	+4.66	100.00	6.76	+3.38	99.96	6.02	+14.75	100.00	11.06	+4.42	100.00	5.30

Chronological Age (years)	Central Incisors			Lateral Incisors			Canines			1st Pre Molars			2d Pre Molars			1st Molars			2nd Molars		
	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]	$\frac{x-m_x}{\sigma_x}$	% with erupted tooth[1]	tooth age[2]
Girls, Upper Jaw																					
4.5	− 3.42	.03	.00	− 4.10	.00	.00	−5.16	.00	.00	−4.57	.00	.00	−3.95	.00	.00	− 2.91	.18	.00	−5.64	.00	.00
5.5	− 2.15	1.58	.00	− 2.98	.14	.00	−4.37	.00	.00	−3.75	.01	.00	−3.33	.04	.00	− 1.49	6.81	.02	−4.93	.00	.00
6.5	− .89	18.67	.08	− 1.85	3.22	.01	−3.58	.02	.00	−2.93	.17	.00	−2.70	.35	.00	− .06	47.61	.26	−4.21	.00	.00
7.5	+ .38	64.80	.49	− .73	23.27	.12	−2.80	.26	.00	−2.11	1.74	.01	−2.08	1.88	.01	+ 1.37	91.47	.99	−3.50	.02	.00
8.5	+ 1.65	95.05	1.32	+ .39	65.17	.56	−2.01	2.22	.01	−1.30	9.68	.06	−1.45	7.35	.05	+ 2.80	99.74	1.96	−2.79	.26	.00
9.5	+ 2.91	99.82	2.30	+ 1.52	93.57	1.38	−1.22	11.12	.07	− .48	31.56	.25	− .83	20.33	.18	+ 4.23	100.00	2.96	−2.07	1.92	.01
10.5	+ 4.18	100.00	3.30	+ 2.64	99.59	2.35	− .43	33.36	.28	+ .34	63.31	.72	− .20	42.07	.49	+ 5.66	100.00	3.96	−1.36	8.69	.06
11.5	+ 5.44	100.00	4.30	+ 3.76	100.00	3.35	+ .34	63.31	.75	+1.16	87.70	1.49	+ .43	66.64	1.04	+ 7.09	100.00	4.96	− .64	26.11	.22
12.5	+ 6.71	100.00	5.30	+ 4.89	100.00	4.35	+1.14	87.29	1.53	+1.98	97.61	2.43	+1.05	85.31	1.80	+ 8.51	100.00	5.96	+ .07	52.79	.61
13.5	+ 7.97	100.00	6.30	+ 6.01	100.00	5.35	+1.93	97.32	2.46	+2.80	99.74	3.42	+1.68	95.35	2.72	+ 9.94	100.00	6.96	+ .79	78.52	1.28
14.5	+ 9.24	100.00	7.30	+ 7.13	100.00	6.35	+2.72	99.67	3.46	+3.62	99.99	4.42	+2.30	98.93	3.69	+11.37	100.00	7.96	+1.50	93.32	2.14
15.5	+10.51	100.00	8.30	+ 8.26	100.00	7.35	+3.50	99.98	4.45	+4.44	100.00	5.42	+2.93	99.83	4.69	+12.80	100.00	8.96	+2.21	98.64	3.10
16.5	+11.77	100.00	9.30	+ 9.38	100.00	8.35	+4.29	100.00	5.45	+5.26	100.00	6.42	+3.55	99.98	5.68	+14.23	100.00	9.96	+2.93	99.83	4.10
17.5	+13.04	100.00	10.30	+10.51	100.00	9.35	+5.08	100.00	6.45	+6.08	100.00	7.42	+4.18	100.00	6.69	+15.66	100.00	10.96	+3.64	99.99	5.10
Girls, Lower Jaw																					
4.5	− 1.82	3.44	.01	− 3.51	.00	.00	−5.45	.00	.00	−4.60	.00	.00	−3.88	.00	.00	− 1.86	3.14	.01	−4.77	.00	.00
5.5	− .68	24.83	.13	− 2.26	1.19	.00	−4.45	.00	.00	−3.80	.01	.00	−3.29	.05	.00	− .71	23.89	.12	−4.13	.00	.00
6.5	+ .45	67.36	.58	− 1.01	15.62	.07	−3.45	.03	.00	−3.00	.13	.00	−2.71	.34	.00	+ .44	67.00	.57	−3.48	.03	.00
7.5	+ 1.59	94.41	1.42	+ .24	59.48	.42	−2.45	.71	.00	−2.20	1.39	.01	−2.12	1.70	.01	+ 1.59	94.41	1.40	−2.84	.23	.00
8.5	+ 2.73	99.68	2.40	+ 1.49	93.19	1.22	−1.45	7.35	.03	−1.40	8.08	.05	−1.53	6.30	.05	+ 2.74	99.69	2.38	−2.19	1.43	.01
9.5	+ 3.86	100.00	3.40	+ 2.74	99.69	2.19	− .45	32.64	.21	− .60	27.43	.21	− .94	17.36	.16	+ 3.89	99.99	3.38	−1.55	6.06	.04
10.5	+ 5.00	100.00	4.40	+ 3.99	100.00	3.19	+ .55	70.88	.73	+ .20	57.93	.63	− .35	36.32	.42	+ 5.03	100.00	4.38	− .90	18.41	.16
11.5	+ 6.14	100.00	5.40	+ 5.24	100.00	4.19	+1.55	93.94	1.58	+1.00	84.13	1.35	+ .24	59.48	.90	+ 6.18	100.00	5.38	− .26	39.74	.44
12.5	+ 7.27	100.00	6.40	+ 6.49	100.00	5.19	+2.55	99.46	2.55	+1.80	96.41	2.27	+ .82	79.39	1.59	+ 7.33	100.00	6.38	+ .39	65.17	.97
13.5	+ 8.41	100.00	7.40	+ 7.74	100.00	6.19	+3.55	99.98	3.55	+2.60	99.53	3.25	+1.41	92.07	2.46	+ 8.48	100.00	7.38	+1.03	84.85	1.72
14.5	+ 9.55	100.00	8.40	+ 8.99	100.00	7.19	+4.55	100.00	4.55	+3.40	99.97	4.25	+2.00	97.72	3.41	+ 9.63	100.00	8.38	+1.68	95.35	2.63
15.5	+10.68	100.00	9.40	+10.24	100.00	8.19	+5.55	100.00	5.55	+4.20	100.00	5.25	+2.59	99.52	4.40	+10.78	100.00	9.38	+2.32	98.98	3.61
16.5	+11.82	100.00	10.40	+11.49	100.00	9.19	+6.55	100.00	6.55	+5.00	100.00	6.25	+3.18	99.93	5.40	+11.93	100.00	10.38	+2.97	99.85	4.60
17.5	+12.95	100.00	11.40	+12.74	100.00	10.19	+7.55	100.00	7.55	+5.80	100.00	7.25	+3.76	99.99	6.40	+13.08	100.00	11.38	+3.61	99.98	5.60

[1] These values represent the integral of the normal probability curve which describes the age distribution of eruption of the tooth.

[2] These values represent the double integral of the normal probability curve which describes the age distribution of eruption of the tooth. The actual arithmetic values recorded here were obtained from a detailed table giving values of the double integral by 0.01 units of $\frac{x-m_x}{\sigma_x}$, from $\frac{x-m_x}{\sigma_x} = -4.5$ to $+4.5$. Values for $\frac{x-m_x}{\sigma_x}$ greater than $+4.5$ were obtained by adding one post-eruptive tooth year of experience in the mouth to the value immediately above in the table.

$$\int_{-\infty}^{x}\left[\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma_x}e^{-1/2(x-m_x/\sigma_x)^2}dx\right]dx.$$

It is apparent that values of this function constitute only indirect estimates of the average number of post-eruptive years of exposure of a particular type of tooth in the mouths of those children having that tooth erupted. Clearly, the value of the function at any age x may not be precisely equivalent to that which would be obtained from the summation of duration of exposure as actually ob-

TABLE 3

AVERAGE NUMBER OF PERMANENT TEETH ERUPTED INTO THE MOUTH PER CHILD AND ACCUMULATED POST-ERUPTIVE TOOTH AGE (ACCUMULATED TOOTH YEARS OF EXPOSURE) PER CHILD (EXCLUSIVE OF THIRD MOLARS). UPPER, LOWER AND BOTH JAWS OF BOYS AND GIRLS FOR EACH CHRONOLOGICAL YEAR OF AGE FROM 4.5 TO 17.5. BASED ON AN ANALYSIS OF DENTAL EXAMINATIONS OF 4,416 ELEMENTARY SCHOOL CHILDREN, HAGERSTOWN, MARYLAND

Chronological Age (Years)	Boys						Girls					
	Upper Jaw		Lower Jaw		Both Jaws		Upper Jaw		Lower Jaw		Both Jaws	
	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]	Average number teeth erupted per child[1]	Accumulated post-eruptive tooth age per child[2]
4.5	.00	.00	.02	.00	.02	.00	.00	.00	.13	.04	.14	.05
5.5	.14	.04	.40	.14	.54	.18	.17	.05	1.00	.51	1.17	.56
6.5	1.10	.56	2.23	1.31	3.33	1.87	1.40	.71	3.01	2.45	4.41	3.16
7.5	3.15	2.65	4.58	4.76	7.72	7.41	3.67	3.23	5.05	6.54	8.72	9.76
8.5	5.05	6.80	5.98	10.13	11.03	16.94	5.59	7.91	6.31	12.27	11.90	20.18
9.5	6.54	12.60	6.95	16.57	13.49	29.18	7.17	14.30	7.66	19.19	14.83	33.49
10.5	8.06	19.88	8.46	24.21	16.52	44.09	8.94	22.33	9.67	27.83	18.61	50.15
11.5	9.81	28.80	10.49	33.67	20.30	62.47	10.88	32.22	11.55	38.49	22.42	70.71
12.5	11.65	39.56	12.30	45.11	23.94	84.68	12.46	43.95	12.81	50.70	25.27	94.65
13.5	13.07	51.96	13.39	58.00	26.46	109.96	13.42	56.97	13.53	63.90	26.95	120.87
14.5	13.76	65.45	13.85	71.70	27.61	137.15	13.84	70.62	13.86	77.64	27.70	148.26
15.5	13.96	79.33	13.97	85.63	27.93	164.95	13.97	84.52	13.97	91.56	27.94	176.08
16.5	14.00	93.30	13.99	99.60	27.99	192.90	14.00	98.52	13.99	105.55	27.99	204.06
17.5	14.00	107.28	14.00	113.61	28.00	220.88	14.00	112.52	14.00	119.55	28.00	232.06

[1] These values were obtained by summing and dividing by 100 the appropriate figures from the rows of columns in table 2 marked "Percent of children having tooth erupted."

[2] These values were obtained by summing the appropriate figures from the rows of columns in table 2 marked "Post-eruptive tooth age."

served in a group of children in a seriatim study. On the other hand, it seems reasonable to expect that values derived through the use of this function may serve very useful purposes in connection with studies on the character of dental caries as it may be influenced by length of exposure of teeth in the mouth.

So far as is known tables of the double integral of the normal probability function are not available in the literature.[4] In order to provide data, therefore, it became necessary to develop a table giving estimated values of this integration. Arithmetic values for the double integral were derived by the finite differ-

[4] It is well known that the expression $e^{-1/2\,z^2}$ cannot be integrated by an application of the standard forms of integrals.

ence method[5] from values of the single integral of the probability function given in a standard 7-place table (3). Obviously, the arithmetic values so obtained are not as accurate as those which could be derived from series expansions of the function. The table developed, however, has been carefully checked and is believed correct to 5 decimal places.

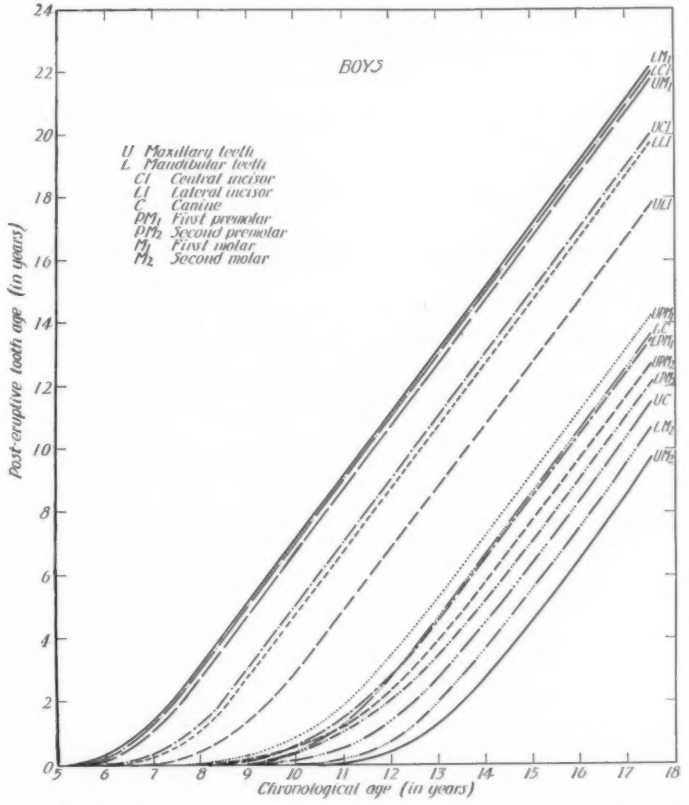

FIG. 1. The relation between chronological age and the average accumulated post-eruptive tooth age of specified corresponding pairs of permanent teeth. 2,232 elementary school boys, Hagerstown, Maryland.

In order to make readily available the calculation of the descriptive constants of tooth age, the appendix table in this paper gives a tabulation of arithmetic values of the double integral of the probability function for each one-tenth unit of the argument, (X), from -4.0 to $+4.0$, where X equals $x-m_x/\sigma_x$.

[5] Areas under the curve of the integral of the normal curve were obtained by straight line approximations for successive intervals of one one-hundredth of a unit of the argument $x-m_x/\sigma_x$.

POST-ERUPTIVE TOOTH AGE TABLES

Through the application of the technique presented above, standard tables for post-eruptive tooth age, specific for each of the several kinds of permanent teeth, have been derived from observations on the teeth of 4,416 grade school children in Hagerstown, Maryland (4). Basic data for the development of these tables consist of the means and standard deviations (Table 1) of the normal frequency functions which have been shown to describe the age distribution of the eruption of the permanent teeth in these children (2). By means of these constants of the normal curve, specific for each type of tooth, table 2 has been constructed to give at yearly intervals of chronological age the following data for each corresponding pair of permanent teeth:

(a) Chronological age expressed as the deviation from the mean age of eruption, divided by the standard deviation of the age distribution of eruption, $x - m_x/\dot{x}$.

(b) Theoretical values for the percentage of children having a particular tooth erupted in the mouth.

(c) The accumulated total post-eruptive tooth age of exposure in the mouth.

In addition to the tabulations for each type of permanent tooth, data on the average number of teeth erupted and their accumulated post-eruptive tooth ages are given in table 3 for the upper and lower jaw and for both jaws together. Study of the tables presented serves to identify several of the major characteristics of tooth age. Graphic illustrations of certain aspects of the relationships of tooth age to other factors are given in figures 1, 2 and 3. In the present paper, mention will be made of only those interrelationships which seem to promise the most usefulness in studies on dental caries in children.

In figure 1 the relationship for boys between chronological age and tooth age for the separate types of permanent teeth is illustrated. The separate curves show the changes, with chronological age, in the post-eruptive tooth age of the specified corresponding pairs of permanent teeth. For example, at the chronological age, 14.5 years, the two permanent lower first molar teeth in boys have, together, an accumulated tooth age in the mouth of approximately 16 years;[6] at the same chronological age (14.5 years) the two lower second molars have a mouth experience of slightly less than 4 years. From this graph, and from table 3, the fact may be derived, also, that the accumulated tooth age of the corresponding lower first molars at a chronological age of 8.5 years is approximately equal to the accumulated tooth age of the lower second molars at a chronological age of 14.5 years. Through the use of these data it is possible to make quantitative comparisons of caries experience of the teeth in terms of specific tooth age rather than chronological age. Results[7] of the application of this concept to the study of dental caries indicate that important characteristics of the constitution of the caries problem may be revealed through expressing caries experience in terms of post-eruptive tooth age.

[6] Table 3 shows that each lower first molar has accumulated in the mouth a total of 8.06 years; both lower first molars, therefore, have a post-eruptive tooth age of 16.16 years.

[7] To be published in a later paper in this series of studies.

In figure 2 the accumulated years of exposure of all teeth in the mouth is related to chronological age. The graph illustrates the fact (explained by the observation that the permanent teeth erupt earlier in girls than in boys) that at each chronological year of age the permanent teeth of girls have a higher accumulated tooth age than those of boys. This graph and table 3 show, for example, that at age 14.5 years all the erupted permanent teeth of boys have accumulated a total of 137.15 years of exposure in the mouth while the erupted

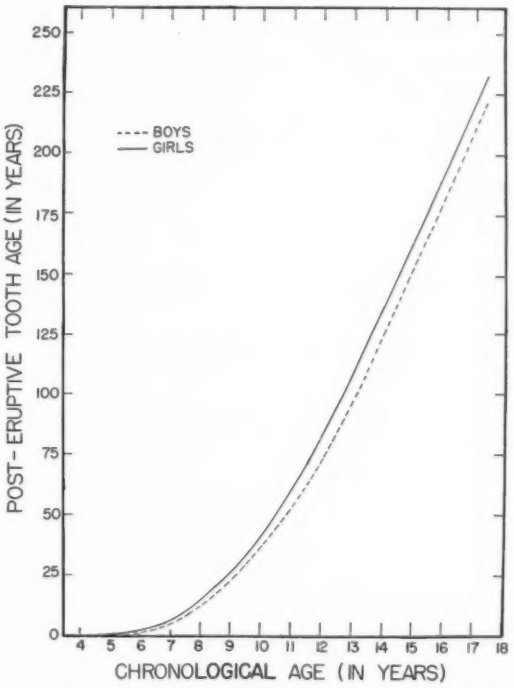

FIG. 2. The relation between chronological age and the average accumulated post-eruptive tooth age of all permanent teeth (exclusive of third molars). 4,416 elementary school children, Hagerstown, Maryland.

permanent teeth of girls have accumulated 148.26 years of mouth exposure. This difference, between the two sexes, in exposure of the teeth to the environment of the mouth is, at this age, approximately 8 percent higher for the girls. Since it is well known that girls have a higher caries experience than boys of the same age, it becomes of interest to point out that this difference in caries experience *may* be related in part to tooth age.

Figure 3 shows the results of relating, graphically, the average number of teeth erupted and accumulated post-eruptive tooth ages.[8] It is apparent from

[8] This graph was obtained by plotting columns 6 and 7 and columns 12 and 13 given in table 3.

an inspection of this graph that the accumulated tooth age of boys and girls *having the same number of permanent teeth in the mouth* is very similar during the major portion of the period in which the permanent teeth erupt. The crossing of the curves, and the higher values of tooth age for boys during the period when 10 to 25 teeth are present in the mouth, indicate interesting differences between the two sexes with respect to exposure of the teeth in the mouth. No detailed

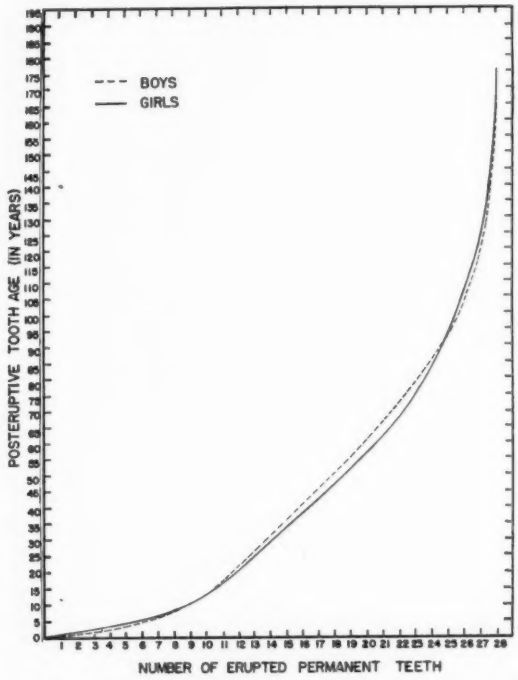

FIG. 3. The relation between average number of permanent teeth erupted and their accumulated post-eruptive tooth age. 4,416 elementary school children, Hagerstown, Maryland.

analysis of these differences and relationships will be given here but it seems reasonable to conclude that further detailed study of tooth age and its relation to caries experience is warranted.

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GROWTH AND DIFFERENTIATION OF ROOTS IN SULPHURETTED SPRING WATERS

By

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The hot sulphuretted waters of certain springs in the Pyrenees mountains have been known from the Roman period to promote the healing of wounds. A military hospital was erected by Louis XIV at Barèges, in order that his wounded soldiers could avail themselves of bathing facilities. That hospital is being run to this day, but research carried on there failed so far to explain the curative property of those waters.

We, therefore, started experiments with seedling plants for the purpose of correlating cell differentiation and cell enlargement with the action of different thermal waters.

The following table shows certain physical properties of the waters from four springs.

Springs	pH	"sulfhydrometric degree"	Radio activity
Raymond	9.5	1.8	0.470
Nouvelle	10.3	8.7	0.753
St. Roch	10.3	12.2	0.610
Tambour	10.3	12.7	0.454

It is well known that root elongation depends upon active division of meristematic cells at the root tip, and that most of the root elongation takes place above the meristematic tissue, where the post meristematic cells elongate previous to differentiating into groups of cells which can be histologically classified into "tissues."

In connection with these studies it should be noted that Gola ('02) first noted the presence of $-SH$ in the meristematic region of root-tips. This was confirmed by Hopkins ('21) and Hammett ('29). The latter extended the finding into a demonstration that $-SH$ is a significant participant in cell increase in number; even to acting as a stimulus to woundhealing (Hammett and Riemann '31). White ('33) also obtained enhancement of proliferation in plant tissue cultures with $-SH$. Furthermore Went, Thimann, and others have shown that optimal doses of the naturally occurring auxin or of the synthetic hetero-auxin enhance cell elongation by increasing the elasticity of the cell wall. Proper doses of these growth substances may even enhance cell division.

In series I (July 1 to 12, 1936) seedlings of *Lens esculentum* were grown in Petri dishes, on absorbent cotton moistened with tap water, or comparatively, with thermal sulphuretted waters from three hot springs.

In series II (July 19 to 26, 1936) seedlings were grown on absorbent cotton, in

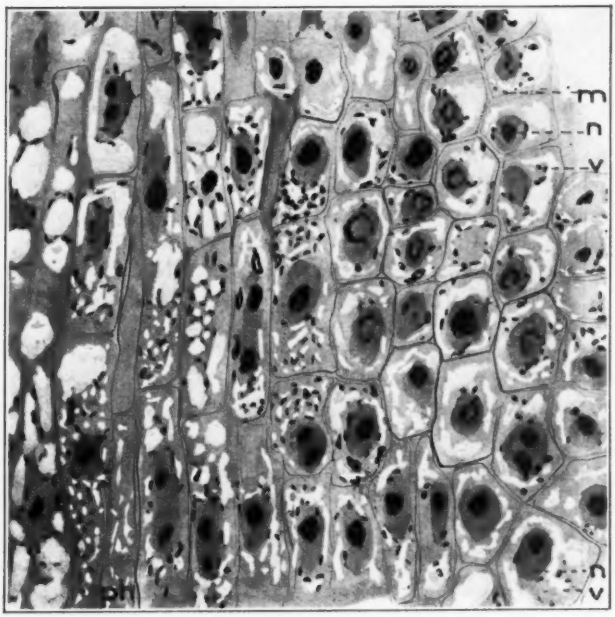

FIG. 1. Longitudinal section through post meristematic tissues at root tip of *Lens esculentum*, germinated in "Tambour" water; *n*, nucleus; (nucleolus pictured black inside); *m*, mitochondria; *v*, vacuole; *ph*, phloem-tissue (the only region where cell elongation is evident).

LENGTHS OF ROOTS OF *Lens esculentus* GROWN IN VARIOUS WATERS
(lengths recorded in millimeters)

		Tambour		St. Roch.		Nouvelle		Ramond		Tap Water	
I	II	I	II	I	II	I	II	I	II	I	II
July											
1	19	2	1	1	2	4	2	5	3	3	3
2	20	7	6	11	5	11	5	14	10	5	3
3	21	12	8	13	10	13	10	18	16	5	
4	22										
5	23										
6	24	20	10	25	30	40	30	40	30	20	15
7	25										
8	26	30	15	40	30	45	30	60	30		20
9		35		45		65		70			
10		60		65		110		110			
11				65		120		120			
12		70				125		125			

test tubes, at room temperature. On account of restricted aeration the seedlings in this lot grew less than these in series I.

The water flowing from the hottest spring (Tambour), which is also the richr est in sodium polysulphide, inhibits root elongation (fig. 1) whereas the wate-

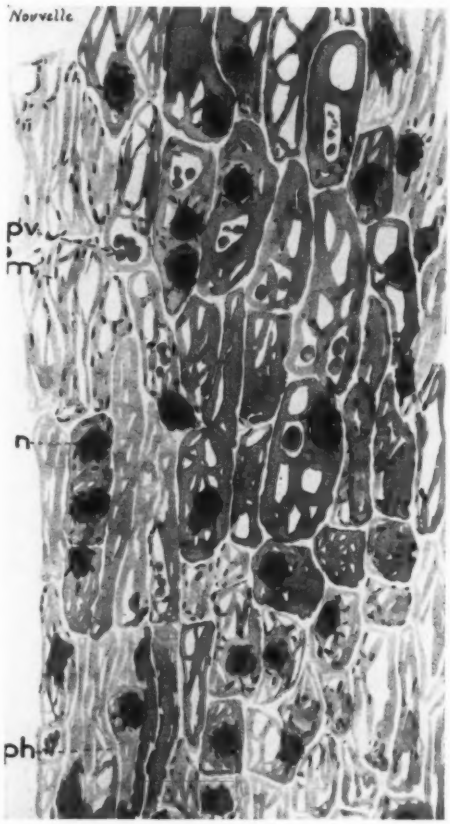

FIG. 2. Homologous tissues of root tip from *L. esculentum* germinated in "Ramond" water; the nuclei (*n*) assume a star shaped appearance due to compression by swelling vacuoles (*v*); smaller vacuoles show as empty spaces (pictured white within the greyish cytoplasm) but larger vacuoles show vacuolar precipitates (*pv*) (featured as dark grains); *m*, mitochlondria; *ph*, phloem.

flowing from the springs, "Nouvelle," "St. Roch" or "Raymond" which we may consider as "diluted Tambour" water (figs. 2 and 3) or the water from the Tambour spring itself, when diluted with an equal volume of tap water, enhances root elongation, when compared to the checks, germinated in tap water.

From the data, we may assume that, growing in the "Ramond" or "Nouvelle" water, which enhances growth, seedlings may develop a root 150 mm. long in the Petri dish, or 200 mm. long in the test tubes.

From the assumed maximum A, we calculated for each successive day $\alpha = A - y/y$ where y is the measured length of root for each seedling in each type of water.

Plotting α against t from $t = 2$ days to $t = 13$ days results in a line which can be adjusted to a straight line of slope $m = -0.40$ for the seedlings grown in Ramond and Nouvelle waters, and to a straight line of slope $m = -0.34$ for the seedlings grown in the "Tambour" water.

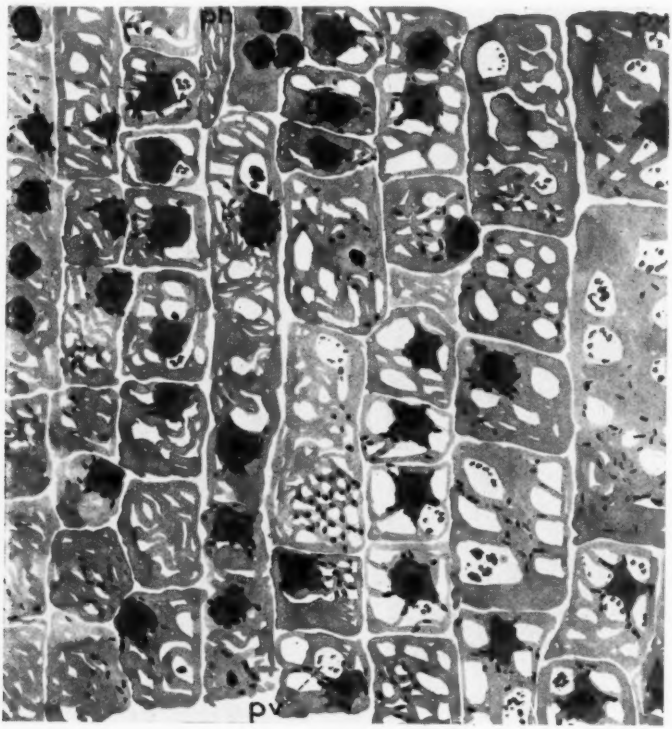

FIG. 3. Homologous tissue from root tip of *L. esculentum* germinated in "Nouvelle" water (same lettering).

Plotting log y against log t results in a line which can be adjusted to a straight line for seedlings grown in the "Ramond" or "Nouvelle" waters, whereas the line shows a definite break after the 10th day, for seedlings grown in Tambour water.

In root tips, which grow very slowly in the Tambour water, post meristematic cells retain their meristematic condition; each cell remains nearly isodiametric; most of its volume is occupied by the spherical nucleus (pictured dark grey (n) fig. 1) with a large nucleolus (pictured black in fig. 1). The nucleus is surrounded

by dense cytoplasm, containing mitochondria (*m*, fig. 1) which outline the contour of the numerous small thread-like vacuoles (*v*). Many mitochondria differentiate into club-shaped amyloplasts, by storing starch grains. Those starch grains proceed from a condensation of glucids which fail to be used up for growth, and we emphasize that such a starch storage is the significant result of growth inhibition, whatever its cause.

Roots growing rapidly in the "Nouvelle" (fig. 2) or "St. Roch" waters, on the contrary, show rapidly elongating post-meristematic cells, wherein the vacuoles swell because the vacuolar contents actively imbibe water. The lower part of fig. 2 shows meristematic cells at a level in the root tip where phloem tissues (*ph*) first begin to differentiate, corresponding to the lower level of fig. 1 where phloem tissue (*ph*) can be clearly recognized.

The upper part of fig. 1 shows cells nearly similar to those at the bottom, while fig. 2 shows all transitions from isodiametric cells at the bottom to the elongated cells at the top.

The three longitudinal sections 1, 2, 3 were drawn with the aid of the camera lucida, at the same scale, using a Leitz 2 mm. objective and ×20 eye piece.

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THEORY OF DIFFUSION FORCES IN METABOLIZING SYSTEMS

By

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The biologist is necessarily concerned with liquid systems, and very often with liquid systems which display certain types of non-equilibrium activity such as chemical reaction, diffusion of dissolved materials, conduction of heat, etc. Such liquid or near liquid systems together with more solid structures such as membranes are, from the point of view of biophysics, the essential constituents of living matter; as has been emphasized by N. Rashevsky in adopting as a prototype cell model a metabolizing liquid system which, through its bounding membrane, exchanges dissolved substances with its external environment (Rashevsky, 1938).

It is seen, therefore, that the so-called *transport phenomena* of diffusion, heat conduction, etc., in solutions are of considerable importance for the biologist. The present paper is concerned primarily with diffusion, and in particular with the mechanical force exerted by a diffusing substance upon particles past which it is moving.

Such forces have been employed by Rashevsky in a number of investigations on the conditions for mechanical instability of spherical cells, and have been referred to by him in discussing such topics as the distribution of colloidal and other particles in living cells, the branching growth of nerve fibers, form development in embryonic growth, etc. (Rashevsky, 1938). Observations have shown the tendency for a moving solute to drag another along with it, as has been noted on a number of occasions by McBain and his co-workers (McBain and Dawson, 1934); and this phenomena of *diffusion drag* has been used by them in discussing a topic in spontaneous emulsification (McBain and Woo, 1937). Some observations in centrifuge experiments (Pederson, 1936) are regarded by McBain and Woo as also involving this phenomena (cf. also Brown, 1918).

Rashevsky's estimate for the value of this drag force, though since improved for certain particle shapes such as spheres and cylinders (Young, 1936) and ellipsoids of revolution (Young and Reiner, 1937), is based on rather rough considerations, while McBain's discussion is qualitative. Since the magnitude of these diffusion forces in living systems apparently may be (Rashevsky, Chapter XII) at least as great as that of the surface tension and electrical forces usually discussed as operative in colloidal behavior, it is of importance to consider their origin and calculation in some detail.

This will be done in the following pages. The work falls naturally into two parts: the first is a kinetic collision treatment appropriate for the action of one solute upon another, while the second is a hydrodynamical treatment for a solute moving past a comparatively large particle which disturbs the diffusion

field in its immediate neighborhood. These correspond precisely to the extreme cases distinguished by Epstein in discussing Stoke's law for the force on a sphere falling through a gas (Epstein, 1924). In the second section it will appear that Fick's law of diffusion omits certain viscosity terms which are important near immersed particles and which are primarily responsible for the diffusion drag on such particles.

It may be noted that the first section corresponds more nearly to the case represented by McBain's articles; while the second part more closely represents Rashevsky's considerations. In the latter it will appear that the calculations of Rashevsky as amended by Young and Reiner do yield an approximation which will usually be very close to the correct value for the force.

I. KINETIC TREATMENT

In this section will be considered the mechanical effect of a moving solute upon particles comparable in size to the solute molecules themselves; i.e., typically, of one solute upon another. It will not be possible to give a complete theoretical treatment because the theory of the liquid state of matter is both too little advanced and far too complicated to permit a rigorous discussion of transport phenomena in solutions. However a certain amount of progress can be made on the basis of the Van't Hoff conception of solutions according to which a (dilute) solute is to be regarded as a (nearly perfect) gas at the temperature of the solution. Diffusion is thus to be pictured as the motion of a gas (the solute) through another fluid (the solvent) which impedes its progress.

For definiteness in what follows attention will be confined to a case of three substances: a moving solute (No. 1), a stationary solvent (No. 2), and a stationary solute (No. 3). It is desired to find the force or drag exerted on the type 3 particles when the type 1 particles have an assigned mass motion. If it were not for the presence of the solvent this problem could be handled by known, although complicated, methods of the kinetic theory of gases. The introduction of the solvent brings in all the complexities of liquid structure, and it is at present simply not possible to try to take these into account.

About the simplest way one could hope to proceed would be to treat the solvent as a third perfect gas, carry out the calculations, and then in the result increase the density of the solvent gas to that of the actual liquid. Even this in general presents difficulties of computation which were for years unsolved. Only in comparatively recent times has the work of Chapman and Enskog provided a means of handling even the case of the two perfect gases. An idea of the complexity of the calculations may be gained by looking at the work of Chapman on the two-gas case (Chapman, 1918).

There is, however, a case discovered by Maxwell for which the calculations may be made in a comparatively simple manner. He has shown (Maxwell, 1866) that if the particles of the different kinds of gases repel each other according to the inverse fifth power of the distance between them,

$$\text{force}_{ij} = K_{ij}/r^5, \tag{1}$$

then the drag exerted by one component moving through another is proportional to the product of their densities and the difference in their mean velocities. Thus the vector force exerted on unit volume of gas i by gas j is

$$\boldsymbol{F}_{ij} = k_{ij}\rho_i\rho_j(\boldsymbol{v}_j - \boldsymbol{v}_i), \tag{2}$$

where

$$k_{ij} = 2.66[K_{ij}/(m_i + m_j)m_im_j]^{1/2}. \tag{3}$$

In the above m is the mass of a particle, ρ the mass density, and $\boldsymbol{v}$ is the mean velocity.

Thus in a system composed of maxwellian particles* the following result is true:

(I) *The average force exerted on a particle by a stationary solvent vanishes with the average velocity of the particle.*

It seems reasonable that this may hold in actual systems as well, and accordingly (I) *will be assumed true* in what follows. This assumption will be further considered in the second section of this paper.

In the Maxwellian case the force beween component gases depends only on their mean velocities, and not upon the way in which the velocities of the individual molecules are distributed about this mean value. In general, however, this is not the case, and in order to calculate the drag force on the basis of any other molecular model it would be necessary to know the velocity distribution functions for the solute molecules. But these functions are affected by the solvent in a way that can not be determined without bringing in a theory of liquids; in other words they at present can not be determined at all.

It is a simple matter to compute the drag which would result from various assumed distribution functions. A common assumption in this connection is that the particle velocities have the Maxwell equilibrium distribution about the mean velocity (Cunningham, 1910; Epstein, 1924; Loeb, 1934, p. 550). If the mean velocity is small compared with the speed of thermal agitation, and if the molecules collide as rigid elastic spheres, the average drag on a type 3 particle is (Loeb, p. 551):

$$f_{31} = \frac{8}{3}\sqrt{2\pi}\sigma_{13}^2\rho_1\sqrt{\frac{m_3}{m_1 + m_3}}\sqrt{\frac{kT}{m_1}}v_1 \tag{4}$$

where σ_{13} = sum of radii of the two type spheres, and k is the Boltzman constant per molecule. Denoting by U_1 the mass diffusion rate $\rho_1 v_1$, this may be written as

$$f_{31} = 6.7\sqrt{\frac{kT}{m_1}}\sqrt{\frac{m_3}{m_1 + m_3}}\sigma_{13}^2 U_1. \tag{5}$$

The quantity $\sqrt{kT/m_1}$ is proportional to the root mean square speed of thermal

* Enskog has shown (Jones, 1923, p. 32) that the Maxwell case of inverse fifth power molecules is equivalent to one in which the molecules behave as rigid spheres, except that their diameter for a collision is inversely proportional to the square root of the relative velocity of the approaching molecule.

agitation of the type 1 particles. Types of collision other than perfectly elastic are discussed by Epstein, and result in changing somewhat the value of the numerical coefficient in (5).

In the case of Maxwellian particles there are no definite radii, and hence (2) can not be immediately compared with (5). However, Jeans has shown (Loeb, p. 220) that the average distance of closest approach in collisions is

$$\sqrt[4]{K_{ij}/4kT}. \tag{6}$$

If this be taken to define what is meant by σ_{13}, then expression (2) gives

$$f_{31} = 5.32\sqrt{\frac{kT}{m_1}}\sqrt{\frac{m_3}{m_1+m_3}}\,\sigma_{13}^2 U_1, \tag{7}$$

which differs from (5) only in the numerical factor.

Expression (5) might be amended by utilizing Chapman's work to find the correct distribution functions for a two-gas rigid sphere system, but it is hardly worth while to do this because ignoring the effect of the solvent still leaves an unknown error in the results. However, an indication of the sort of result which would be gotten is readily obtained in case the thermal speed of the type 3 particles is much less than that of the type 1 particles; i.e., if $m_3 \gg m_1$. For if the No. 3 particles be regarded as stationary, the No. 1 velocity distribution function is available in the work of Lorentz on conduction of heat and electricity in metals (Lorentz, 1909, p. 266). For such a distribution it can be shown that for a system varying only in the x-direction

$$f_{31} = 5.9\sqrt{\frac{kT}{m_1}}\,\sigma_{13}^2\,\frac{\frac{\rho_1}{kT}X_1 - \frac{1}{m_1}\frac{d\rho_1}{dx} - \frac{\rho_1}{m_1 T}\frac{dT}{dx}}{\frac{\rho_1}{kT}X_1 - \frac{1}{m_1}\frac{d\rho_1}{dx} - \frac{\rho_1}{2m_1 T}\frac{dT}{dx}}\,U_1, \tag{8}$$

where X_1 is the external force per unit mass. The presence of dT/dx illustrates the fact that diffusion depends upon the temperature gradients as well as on the density gradient; a fact pointed out by Chapman and referred to by him as *thermal diffusion*. If the temperature variation may be neglected, which is probably the case for some biological systems, especially single cells, because of the fairly high thermal conductivity of aqueous solutions, this reduces to

$$f_{31} = 5.9\sqrt{\frac{kT}{m_1}}\,\sigma_{13}^2 U_1, \tag{9}$$

which is intermediate between (5) and (7) for $m_3 \gg m_1$.

These various expressions for the force agree fairly well, and one thus concludes that

(II) *If assumption* (I) *is made and the effect of the solvent on the velocity distributions of the solute particles be ignored, then the average drag on a type 3 particle arising from the diffusion of solute No. 1 is given by*

$$f_3 = \lambda \sqrt{\frac{kT}{m_1}} \sqrt{\frac{m_3}{m_1 + m_3}} \sigma_{13}^2 U_1, \tag{10}$$

where the value of λ *is somewhere about 5 to 7.*

This is as far as the discussion will be carried in the present paper. From what has been said it will be clear that a more rigorous treatment is a matter of considerable difficulty. It is to be hoped that experiments will be made to provide a test of the present tentative result (10), but until this is done it hardly seems worthwhile to attempt theoretical improvements.

It may be of interest to note some numerical results of treating a liquid as a dense but perfect Maxwellian gas. The following calculation may perhaps be indicative. Considering two kinds of particles as follows:

No. 1 (water): mol. wt. $= 20$
mol. diam. $= 2 \times 10^{-8}$ cm.
density $= 1$ g./cm.^{3}

No. 2 (solute): mol. wt. $= 50$
mol. diam. $= 3 \times 10^{-8}$ cm.

and determining K_{12} from (6), gives for the diffusion coefficient of the solute about $(1/3)10^{-3}$ cm./sec. Data on comparable solutes (Handbook of Chemistry and Physics, 21 edition, Cleveland, 1936, p. 1238) gives about 10^{-5} cm./sec., so that the diffusion coefficient calculated from such a simple model for an aqueous solution is about 30 times too large.

This means that the calculated water-solute drag is only about 1/30 of its actual value. The calculated value might be improved by taking account of attractive forces which would increase the apparent molecular diameter *for collisions* in a manner which has been described by Sutherland (Loeb, pp. 224; 563).

This will not be discussed further here.

II. HYDRODYNAMICAL TREATMENT

The process of diffusion plays an important role in biophysical investigations. For the most part such studies have employed the formula given by Fick as an analogy to Fourier's law for the flow of heat and Ohm's law for the flow of electricity; namely that the (mass) rate of flow of a dissolved substance is proportional to the gradient of its concentration, i.e.,

$$\boldsymbol{U} = \rho \boldsymbol{v} = -D \nabla \rho, \tag{11}$$

where D, the coefficient of diffusion, depends upon the composition of the system at the point in question.

It is well known that this formulation requires modification in certain cases; for example, if an external force field acts upon the diffusing particles as in ionic diffusion, or if there are temperature gradients in the system, or convective currents in the solvent. In the following pages diffusion will be considered from the standpoint of the general equations of motion of hydrodynamics; and it

will appear that, in addition to the above mentioned items, Fick's equation omits certain viscosity considerations. These are not important for "smooth" types of flow, such as in the systems employed in experimental set-ups to test Fick's law and measure the diffusion coefficient; but in other cases they may completely alter the nature of the flow, such as that about small bodies in the diffusion stream.

The fundamental equations of hydrodynamics are the so-called Navier-Stokes equations given (Lamb, 1924, p. 546) by

$$\begin{aligned}
\rho \frac{Du}{Dt} + \frac{\partial p}{\partial x} &= \rho X + \eta\left[\frac{1}{3}\frac{\partial \theta}{\partial x} + \nabla^2 u\right] \\
\rho \frac{Dv}{Dt} + \frac{\partial p}{\partial y} &= \rho Y + \eta\left[\frac{1}{3}\frac{\partial \theta}{\partial y} + \nabla^2 v\right] \qquad (12) \\
\rho \frac{Dw}{Dt} + \frac{\partial p}{\partial z} &= \rho Z + \eta\left[\frac{1}{3}\frac{\partial \theta}{\partial z} + \nabla^2 w\right].
\end{aligned}$$

Herein D/Dt denotes the "convective" differential operator

$$\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z};$$

ρ is the density of the moving fluid, η is its viscosity coefficient, and $u\ v\ w$ are its component mean velocities along $x\ y\ z$; and

$$\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.$$

In a moving viscous fluid the pressure at a point is in general different in different directions. However, its arithmetic mean in any three mutually perpendicular directions is the same, and this mean is denoted by p. $X\ Y\ Z$ are the components of external force per unit mass of fluid.

These equations may, with suitable assumptions, be derived either on the basis of a continuous fluid picture (Lamb, p. 541) or from the kinetic theory of gases (Bulletin National Research Council, 1931, p. 495); so that they may be applied to either case. To them are adjoined the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0, \qquad (13)$$

and some relation involving the density, which is usually taken to depend only on the pressure (Lamb, p. 611; Prandtl-Tietjens, 1934, vol. I, p. 111), i.e.,

$$\rho = \rho(p). \qquad (14)$$

The terms $X\ Y\ Z$ for each component fluid arise from forces external to the system as a whole, such as gravity; from short range interaction with other components of the system, which depends only on the part of the system in the

immediate neighborhood of the point in question; and from long range, such as electrical, interaction which may be contributed to by all parts of the system. External and long range forces will be ignored for the present, and it will be assumed that

(III) *The short range interaction between two fluid components can be satisfactorily represented by a resistance drag proportional to their relative velocity.*

This includes (I) as a special case, follows from (2) for the case of a Maxwellian system, and is consistent with (II). It has been employed by Jaeger (Jaeger, 1899) and Einstein (Loeb, p. 405) who obtained respectively, expressions for the mobility and diffusion coefficients by treating the moving particles as spheres to which Stoke's law is applicable; and it is required by the conventional equations for free and forced diffusion. In addition, (III) is about the only assumption which is mathematically tractable.

Confining attention to a moving solute (No. 1) and a stationary solvent (No. 2), it is therefore assumed that

$$\begin{aligned} X_1 &= -bu \\ Y_1 &= -bv \\ Z_1 &= -bw. \end{aligned} \tag{15}$$

Since the short range interactions will be equal and opposite within any volume element, it follows that

$$\rho_2 X_2 = -\rho_1 X_1 = b\rho_1 u; \quad \text{etc.} \tag{16}$$

The quantity b is strictly proportional to ρ_2 in a Maxwellian system, and such proportionality is consistent with (II). Since ρ_2 is substantially constant in a liquid system *it will be assumed that b is constant.* This assumption is required by the constancy of the diffusion and mobility coefficients.

Thus in the absence of long range or external forces the equations of motion of a dissolved substance in an incompressible stationary solvent are, in vector notation with $\boldsymbol{v} = (u, v, w)$,

$$\nabla p = -\rho \frac{D\boldsymbol{v}}{Dt} - b\rho\boldsymbol{v} + \eta(\tfrac{1}{3}\nabla\theta + \nabla^2\boldsymbol{v}). \tag{17}$$

Regarding a dilute solute as a perfect gas gives for (14):

$$\rho_1 = \frac{m_1}{kT} p_1. \tag{18}$$

In the steady state, at constant temperature, and with the viscous and inertial (the last three terms of D/Dt) terms neglected, (17) then becomes

$$\frac{kT}{m_1} \nabla\rho_1 = -b\rho_1\boldsymbol{v}; \tag{19}$$

which is the same as (11) with

$$D = \frac{kT}{bm_1} \cdot \tag{20}$$

Hence Fick's law may be regarded as obtained, under certain conditions, by dropping the inertial and viscous terms in the hydrodynamic equations. The existence and constancy of b are related to those of D as mentioned above. It has been noted by Rayleigh that Ohm's law for the motion of electricity could be similarly obtained by assuming a resistance force proportional to the velocity (Rayleigh, 1896, vol. II, p. 6).

In general the approximations made in obtaining Fick's law from (17) will be valid in more or less "open" regions of liquid systems. The same, however, will not be true near material boundaries such as the surfaces of immersed particles, for here the flow changes rapidly over short distances and thus the derivative terms $\nabla^2 u$, etc. may become relevant. The reason for this is found in the boundary conditions to be satisfied at such surfaces.

It is apparent that a moving fluid must have zero normal velocity at the surface of a stationary body which it does not penetrate; and it is more or less a matter of observation that a fluid moving slowly past a regular shaped obstacle tends to cling to it so that the tangential velocity at the surface is zero as well. This is borne out by certain theoretical results in hydrodynamics. If only zero normal velocity is required, the conditions of the problem can be satisfied by a so-called potential flow which makes the viscous terms very small or zero, but has a sizeable tangential component at the body surface. It is then found that the resulting drag on an isolated body is zero, quite contrary to experience (Prandtl, II, 108; I, 260) (Lamb, p. 641) (Rayleigh, 1916).

On the other hand, if no-slip boundary conditions are imposed it is necessary to retain the viscous terms in order to get a solution at all. Since the complete Navier-Stokes equations are hard to handle, progress can usually be made only by dropping the inertial terms. In this way Stokes was able to obtain a formula for the drag on a sphere which agrees well with experiment. To a first approximation his result is unchanged by a partial inclusion of the inertial terms as suggested by Oseen (Lamb, p. 577) (Bulletin National Research Council, p. 316).

One thus concludes that an unhampered fluid moving slowly past a body does not slip at the surface. Sometimes other assumptions are made involving a surface coefficient of "sliding friction," (Lamb, pp. 546; 569) (Bulletin National Research Council, pp. 90; 159), but this usually does not make much difference in the results. The important point is that they impose some conditions on the tangential velocity at the surface.

Now the case of diffusion past a body presents essentially the same kind of situation, except that the moving gas is hindered by the presence of the solvent. Since the solvent itself clings to the body surface, there seems no reason to suppose that its presence will encourage slipping on the part of the solute, and hence it will be assumed that

(IV) *The flow of a dissolved substance satisfies no-slip boundary conditions at*

the surface of an immersed body. For simplicity the possibility of a finite coefficient of sliding friction will be ignored.

It can be readily shown that solutions of Fick's equation are like potential flows in that they cannot satisfy no-slip boundary conditions. If D in (11) is constant, the equation of continuity gives in the steady state

$$\nabla \cdot U = - D \nabla^2 \rho = 0,$$

or

$$\nabla^2 \rho = 0. \tag{21}$$

Thus ρ is harmonic and is therefore determined uniquely by assigning its normal derivative over the body surfaces, along with suitable conditions at infinity. The tangential derivative cannot be prescribed in addition, and thus the flow cannot be required to satisfy no-slip boundary conditions. If D is not a constant we have instead of (21):

$$\nabla^2 \rho + \frac{1}{D} \nabla D \cdot \nabla \rho = 0. \tag{22}$$

However, this is still a second order differential equation, and the argument still holds (Prandtl, I, 261). Hence

(V) *Fick's equation is inconsistent with the proper boundary conditions at the surface of immersed bodies.*

The previous discussion leads one to suspect that this difficulty is to be avoided by retaining the viscous terms of the equation of motion. This is further indicated by considering the effect of adding only the inertial terms to Fick's equation. For in this case also, just as in the mechanics of an unhampered fluid, a flow once irrotational remains irrotational and a velocity potential continues to exist (Rayleigh, II, 6). The class of potential flows is thus of rather general importance, just as in usual hydrodynamic theory (Prandtl, I, 125), and these, at least, do not avoid the boundary difficulties of Fick's equation. One thus concludes that

(VI) *The proper boundary conditions at the surface of immersed bodies require retaining the viscosity terms in the equation of motion of a dissolved substance.*

The stress exerted on a surface by a fluid arises partly from the mean pressure of the fluid, and partly from its rates of shear at the surface. Thus, for example, the x component of the stress exerted across the surface of a sphere is p_x, where (Lamb, p. 602)

$$rp_x = - xp + \eta \left[\left(r \frac{\partial}{\partial r} - 1 \right) u + \frac{\partial}{\partial x} (xu + yv + zw) \right]. \tag{23}$$

Herein r and x are measured from the sphere center. Similar expressions are available for any surface.

For a stationary solvent (12) and (16) give

$$\nabla p_2 = b \rho_1 v_1. \tag{24}$$

But no-slip boundary conditions make $v_1 = 0$ at the surfaces of stationary bodies, so that p_2 is constant over such surfaces. The shear rate of the solvent is identically zero, so that

(VII) *A stationary solvent exerts no force or torque on a stationary body.*

This should be compared with (I). Thus the only effect of the solvent is to modify the flow of the solute, and the total stress on any surface is to be found by employing p_1 and v_1 in (23) or some similar expression.

The first estimate of the diffusion drag on a body was given by Rashevsky who dropped the viscous term in (23) and took for the p_1 field that which would exist if the body were removed (Rashevsky, 1936). This makes the calculated body force equal to the drag exerted by the solute on the corresponding volume of solvent in the unperturbed flow. Thus for a body in an otherwise constant-flow field of strength U his result is

$$\boldsymbol{F} = Vb\boldsymbol{U}, \tag{25}$$

where V is the volume of the body.

A better approximation is obtained by modifying the flow to have zero normal component over the body surface, Fick's law still being employed for the space equation. The pressure is then determined by the following equations:

$$\begin{aligned} \nabla p &= -b(\rho \boldsymbol{v}) \\ \nabla \cdot (\rho \boldsymbol{v}) &= 0 \\ \rho \boldsymbol{v} &\to \boldsymbol{U} \quad \text{at} \quad \infty \\ \rho \boldsymbol{v} \cdot \boldsymbol{n} &= 0 \text{ on body surfaces.} \end{aligned} \tag{26}$$

Here $\boldsymbol{n}$ is a vector in the direction of the normal to the surface.

For a single sphere, and with $\boldsymbol{U}$ a constant vector $\boldsymbol{Ui}$ in the x direction, the result (Young, 1936) is

$$\begin{aligned} p &= -bUx - A\frac{x}{r^3} + \text{constant} \\ \rho u &= U + \frac{A}{b}\frac{\partial}{\partial x}\frac{x}{r^3} \\ \rho v &= \frac{A}{b}\frac{\partial}{\partial y}\frac{x}{r^3} \\ \rho w &= \frac{A}{b}\frac{\partial}{\partial z}\frac{x}{r^3}; \end{aligned} \tag{27}$$

where

$$A = bUa^3/2, \tag{28}$$

a being the sphere radius. On the sphere surface the x component of stress is

$$p_x = \frac{3}{2}bUx^2/a, \tag{29}$$

and the total x force is

$$F = \int\int p_x dS = \frac{3}{2} VbU. \tag{30}$$

The result in this case is to insert a 3/2 factor into (25); for other shape bodies the factor varies somewhat.

This result was used by Rashevsky later (1938), together with some additional approximate kinetic considerations, which lead to a formula, differing from (30) by a coefficient of the order of unity, and depending on the nature of the solvent. As will be seen later, the present more rigorous treatment leads to correction terms in (30) which are of a different nature from the correction term used by Rashevsky.

Since it is practically impossible to solve the complete equation (17), the inertial terms will be neglected. This seems a priori plausible in view of the extremely low diffusion rates in liquid systems. For a living cell Rashevsky gives as a typical set of values $\rho v = \frac{1}{3} 10^{-9}$ g./cm.^{2}/sec. at a concentration of $\rho = 10^{-4}$ g./cm.^{3}, so that v is of the order of 10^{-5} cm./sec. (Rashevsky, 1938, chapters VII, X). The inertial terms, $u(\partial u/\partial x)$, etc., are of the second degree in the velocity components, so that their neglection in comparison with first degree terms, $\nabla^2 u$, etc., is more justified the smaller the flow velocity. The validity of such an approximation can be further discussed after the resulting equations are solved.

In the Fick's law approximation (26) it makes no difference whether the moving fluid is compressible or not, since the density appears only in the combination $\rho \boldsymbol{v}$, which may be replaced by a single symbol. In (17), however, this is not true. Since the equations are simpler for an incompressible fluid, this case will be treated first. For a steady state flow the equations are

$$\begin{aligned} (\nabla^2 + h^2)\boldsymbol{v} &= \frac{1}{\eta} \nabla p \\ \nabla \cdot \boldsymbol{v} &= 0 \end{aligned} \tag{31}$$

where $h^2 = -b\rho/\eta$. This pair of equations has been studied in connection with sinusoidal oscillations of a viscous fluid (Lamb, pp. 601, 618). For the above mentioned situation of a sphere in an otherwise uniform flow the solution can be written down almost directly from Lamb's discussion of a ball pendulum swinging in an incompressible viscous fluid (Lamb, p. 608). The present boundary conditions are

$$\begin{aligned} \rho u = U,\ v = w = 0 \quad &\text{at} \quad \infty \\ u = v = w = 0 \quad &\text{at} \quad r = a, \end{aligned} \tag{32}$$

and the solution is

$$p = -bUx - A\frac{x}{r^3} + \text{constant}$$

$$
\begin{aligned}
\rho u &= U + \frac{A}{b}\frac{\partial}{\partial x}\frac{x}{r^3} + 2Bf_0(hr) - Bf_2(hr)h^2r^5\frac{\partial}{\partial x}\frac{x}{r^3} \\
\rho v &= \frac{A}{b}\frac{\partial}{\partial y}\frac{x}{r^3} - Bf_2(hr)h^2r^5\frac{\partial}{\partial y}\frac{x}{r^3} \\
\rho w &= \frac{A}{b}\frac{\partial}{\partial z}\frac{x}{r^3} - Bf_2(hr)h^2r^5\frac{\partial}{\partial z}\frac{x}{r^3},
\end{aligned} \tag{33}
$$

where

$$
\begin{aligned}
f_n(x) &= \left(-\frac{1}{x}\frac{d}{dx}\right)^n \frac{e^{-ix}}{x} \\
2Bf_0(ha) &= -U \\
A &= bf_2(ha)h^2a^5B.
\end{aligned} \tag{34}
$$

In order that $f_n(hr)$ shall vanish at $r=\infty$, the signs are chosen so that ih is positive (Lamb, p. 607), i.e.,

$$
\begin{aligned}
h &= -i\alpha \\
\alpha &= +\sqrt{b\rho/\eta}.
\end{aligned} \tag{35}
$$

In terms of the original quantities

$$
A = \frac{bU}{2}(a^3 + 3a^2\sqrt{\eta/b\rho} + 3a\eta/b\rho), \tag{36}
$$

of which (28) is the first term. In the limiting case $\eta=0$ it is seen that the pressure field is unaltered by the change in the space equation and boundary conditions. It is, however, readily verified that the shear rates at the sphere surface become infinite as $\eta\to 0$. Thus for small η the pressure field is comparatively stable under a change in boundary conditions, whereas the flow field is violently affected in the immediate neighborhood of the body.

It follows from Lamb's formulae that the last term in (23) gives, for the flow described by (33), a force in the x direction of amount

$$
F_v = 4\pi U(a\eta/\rho + a^2\sqrt{b\eta/\rho}); \tag{37}
$$

while the pressure term contributes

$$
F_p = 2\pi U(a\eta/\rho + a^2\sqrt{b\eta/\rho} + a^3b). \tag{38}
$$

The total force on the sphere is thus

$$
F = F_v + F_p = 2\pi U(a^3b + 3a^2\sqrt{b\eta/\rho} + 3a\eta/\rho), \tag{39}
$$

which reduces to (30) as $\eta\to 0$. In terms of $W=U/\rho$, the actual speed of the moving fluid at great distances, this is

$$
\begin{aligned}
F = {} & 6\pi\eta aW + 6\pi a^2\sqrt{b\eta\rho}W \\
& + 2\pi a^3b\rho W;
\end{aligned} \tag{40}
$$

which reduces to the first term for $b=0$, thus giving Stokes formula for the force on a sphere moving uniformly in a single fluid.

It has been noted that the resulting force agrees with that of (30) for the limiting case $\eta=0$. It remains to see how nearly this is realized in actual cases. The following set of values is suitable for a living cell (Rashevsky, 1938, Chap. V):

M = molecular weight of diffusing substance = 60
$D=10^{-6}$ cm.^{2} min.$^{-1}=(1/60)\ 10^{-6}$ cm.^{2} sec.$^{-1}$
$T=20°$ Cent. or 293° absolute
$\rho=10^{-4}$ g. cm.$^{-5}$.

The viscosity coefficient η is that for *the solute rubbing against itself:* it is not to be confused with the viscosity of the solvent or of the cytoplasm as a whole. Its value is a little doubtful, but on the basis that the solute is to be regarded as a gas it may be taken equal to that for a typical gas, say air, which gives

$$\eta = 2 \times 10^{-4} \text{ g cm.}^{-1} \text{ sec.}^{-1}.$$

As will be seen shortly the precise value for η is not very important anyway.

From these values

$$b = \frac{kT}{mD} = \frac{RT}{MD} = 2.4 \times 10^{16} \text{ sec.}^{-1}.$$

The respective terms in the total force,

$$F = 2\pi U a(a^2 b + 3a\sqrt{b\eta/\rho} + 3\eta/\rho), \tag{39}$$

will be in the ratio of the quantities inside the parentheses. Now

$$\eta/\rho = 2 \text{ cm.}^2 \text{ sec.}^{-1}$$

$$\sqrt{b\eta/\rho} = 2.2 \cdot 10^8 \text{ cm. sec.}^{-1};$$

so that the terms are as

$$a^2 : 2.7 \times 10^{-8} a : 2.5 \times 10^{-16},$$

where a is to be expressed in cm. This is illustrated by the following table giving the ratios of the last two terms to the first for different values of a.

a in cm.	Second term / first	Third term / first
10^{-8}	2.7	2.5
1.58×10^{-8}	1.7	1
5×10^{-8}	5.4×10^{-1}	0.1
10^{-7}	2.7×10^{-1}	0.025
10^{-6}	2.7×10^{-2}	2.5×10^{-4}

Even if η is taken as large as the value for water, i.e., about 10^{-2}, the first term predominates down to about $a=5\times10^{-7}$ cm.

Thus it is seen that for a sphere the Fick approximation (26) gives a good value for the force down to sphere radii of the order of 10^{-7} cm. Since this size is approaching that of the water molecules themselves, this is about as far as one could maintain the assumption of treating the fluids as continuous. The Einstein derivation of the diffusion coefficient does seem to indicate that such an assumption continues to hold fairly well down to molecular size spheres; but one should remember that this is of the nature of an extrapolation.

The above work has been carried out only for a spherical particle, but it seems safe to suppose that the method of (26) will apply to other smooth shape bodies as well. The basic point seems to be that for small η the pressure field, which contributes the dominant force term, is comparatively little affected by the change in boundary conditions and space equations; and this would be expected to hold for other shape bodies as well, at least if they are sufficiently isolated. Whether this would be true or not if particles are close together is a little more questionable, but in a general way it appears reasonable to conjecture that in liquid systems

(VIII) *A good approximation to the force and torque on a body in the diffusion field of an incompressible fluid is obtained by using Fick's approximation with zero normal velocity boundary conditions, and evaluating the stress from the pressure field of the moving substance alone.*

This is subject to the condition that the inertial terms neglected in (31) shall not be too important. Since all the velocity terms in (33) are proportional to U, it is readily seen that for U sufficiently small the inertial terms will be negligible compared to the ones retained. In any actual case this may be investigated by actually evaluating the inertial terms (Lamb, p. 573) and effecting the comparison. A general discussion would be rather lengthy and will not be attempted here.

One further item remains to be discussed, that is the effect of the compressibility of actual solutes. It appears that in the limit $\eta = 0$ this will be of little importance, for the following reason. Whether one employs the Fick approximation, the space equation for a compressible fluid, or the equation for an incompressible fluid, the boundary conditions on the flow field are the same, namely as given in (26). For small η the viscosity terms are important only near the surface S; at fairly short distances (26) practically holds. The only difference between compressible and incompressible fluids appears in the viscosity term in (17), and is thus effective only near S. Judging from the solution obtained in the incompressible case, the pressure field is quite stable under changes in the space equation and has no irregularities. Hence near S the variation in ρ will be smooth, and the difference between $1/\rho$ times the derivatives of $\rho\nu$ and the derivatives of ν (which marks the difference between incompressible and compressible fluids) should not be great. It has already been seen that in practice η appears to be small enough so that conclusions drawn from the limiting case $\eta = 0$ hold very nearly. Hence one surmises that (VIII) may be true also for the diffusion of a compressible fluid.

Without going into detail, the correctness of this may be indicated as follows.

The surmise is that the p and ρv fields given by (33) should satisfy, for $\eta \to 0$, the equations obtained by retaining the complete viscosity term in (17); i.e., equations of the form

$$\frac{\partial p}{\partial x} = -b\rho u + \eta\left(\frac{1}{3}\frac{\partial \theta}{\partial x} + \nabla^2 u\right). \tag{41}$$

From the equation of continuity, $\nabla \cdot (\rho v) = 0$, it follows that

$$\theta = \nabla \cdot v = -\frac{1}{\rho} v \cdot \nabla \rho; \tag{42}$$

and hence that $\partial\theta/\partial x$ involves only the first space derivatives of the velocity components. Similarly, $\nabla^2 u$ in the compressible case differs from its value in the incompressible case only by terms involving the first or zero order derivatives of ρv in (33). At the surface it follows from (31) and (32) that

$$\nabla p = \eta \nabla^2 v. \tag{43}$$

But (33) and (36) show that the pressure gradient remains finite over S, even when η vanishes; so that at $\eta = 0$ terms in $\nabla^2 v$ must become infinite like $1/\eta$. Now a little calculation from (33) indicates that first order space derivatives become infinite only as $1/\sqrt{\eta}$, and are hence at the surface S negligible compared with the second order derivatives. Since near S is the only place where the entire viscosity term is of any importance, the conclusion is that the compressibility may be ignored for η small enough. Thus, if η is sufficiently small,

(IX) *Conclusion VIII holds for the diffusion of compressible solutes as well.*

The question as whether in actual cases η is effectively near enough to zero will not be discussed here. For the numerical values employed above the ratio of $1/\eta : 1/\sqrt{\eta}$ is of the order from 10 to 100, which would seem to suffice for the present conclusion to hold at least roughly. In the further development of the theory a more rigorous treatment of the compressible case will of course have to be given.

SUMMARY

The theory of mechanical forces in diffusion fields, used by N. Rashevsky in the treatment of various mathematico-biophysical problems, is developed here in a more rigorous form than that given by Rashevsky. The expressions derived differ from those of Rashevsky, but justify his conclusions as first approximations.

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GROWTH PHENOMENA OF PINEAPPLE FRUITS

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INTRODUCTION

Data on the growth of pineapple fruits have appeared from time to time in a few publications. No mathematical analysis of such data demonstrating the nature of the growth of these fruits has appeared. In the present paper, values for the volumes and for the weights of whole fruits, for fruitlets and for such sections of the fruits as flesh, shell and core are presented. The object of the paper is to show the nature of the growth of pineapple fruits and the time required for the core, shell and flesh of such fruits to complete growth and reach maturity.

HISTORICAL

Growth studies of fruits have not been as extensive as those of plants. According to Robertson (5), Anderson (1) studied the growth of *Cucubita pepo* at different stages of development and obtained a sigmoid curve by plotting fruit weights against time. Anderson used, for his studies, the same fruit throughout the entire growth. He obtained the weight of this fruit at different intervals by allowing the vine to grow in the open air and the fruit to rest upon cotton wool in the scale-pan of a self-registering balance. The growth curve obtained resembled very closely the curve of an autocatalytic monomolecular reaction as pointed out by Robertson (5).

Sinnott's (7, 8) studies on Cucurbits have shown that there are four processes involved in fruit size differences: (a) cell division, which ceases at about the time of flowering, (b) cell expansion, which is stopped by the formation of the secondary cell wall, (c) attainment of sexual maturity, and (d) development of secondary cell wall. The present authors, reporting on the growth curve of pineapple plants in an earlier paper (6), showed that this curve follows that of an autocatalytic monomolecular reaction. Also, earlier studies by Reed (4) etc., have indicated that the growth curve of other plants was of the same type.

THE PINEAPPLE FRUIT

The pineapple fruit, Figure 1, is known botanically as a sorosis (from the Greek $\sigma\omega\rho\acute{o}s$ = a heap), i.e., a collective or multiple fleshy or pulpy fruit formed by the union of many small flowers which are supported on a central stalk. In a fruit of this kind, the fruitlet is the unit or the individual, and the entire fruit the colony. A pineapple fruit may be composed of a few or many fruitlets, the latter arranged spirally on the stem (central column or core), in the same manner as the leaves are on the stem.

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The different flowers or florets of the pineapple fruit do not open or mature at the same time. Blossoming proceeds spirally up the flower stem from base to apex. It requires from 10 to 28 days for the flowering of all the florets of a fruit, the period being considerably influenced by the number of florets in the fruit.

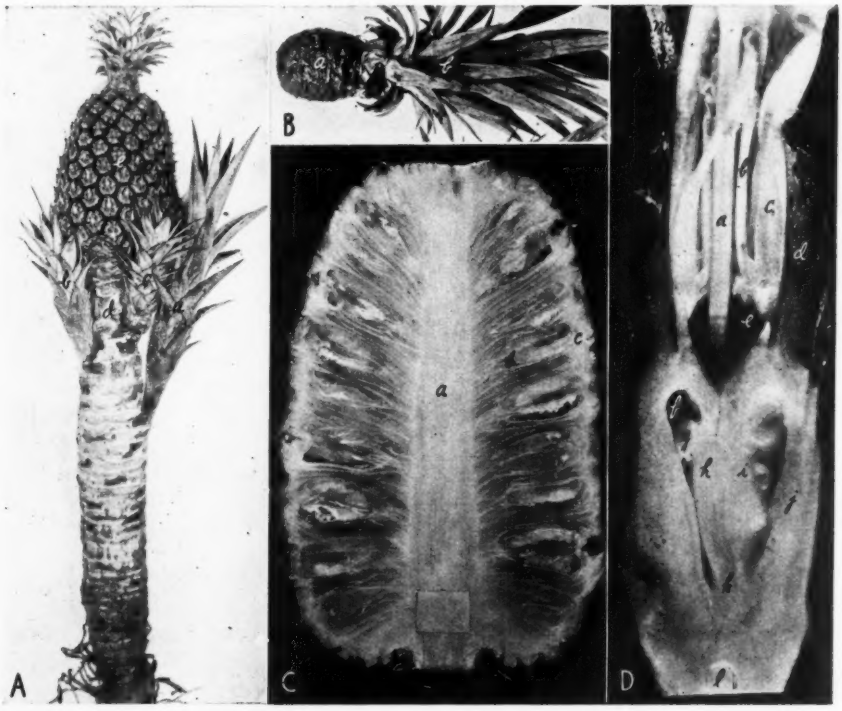

FIG. 1. (A) Pineapple plant with leaves detached and other organs as follows: (a) shoot, (b) hapa, or slip with vestigial fruit almost lacking, (c) slip, (d) peduncle, (e) fruit.

(B) Pineapple slip with (a) *lower* vestigial fruit or slip ball and (b) upper vegetative proliferation.

(C) Ripe pineapple fruit and sections as follows: (a) core, (b) flesh, and (c) shell.

(D) Young pineapple flower (×5). (a) style, (b) stamens, (c) petals, (d) sepals, (e) floral cavity, (f) ovule or seed cavity, (g) placenta, (h) carpel, (i) ovules, (j) pericarp, (k) base or connection point of carpels, (l) base or connection point of fruitlets, (m) bract. (Figures A and B by courtesy of Drs. J. L. Collins and M. B. Linford.)

When the period of receptivity of the pistil is over, the stamens, corolla and style, (Figure 1) wither and gradually dry up. The ovules, if fertilized, become seeds; if not, they gradually become less prominent due to the rapid development of the tissues of the carpels which tend to fill up the ovule cavity.

The Cayenne variety of pineapples, which is grown commercially in Hawaii, is self incompatible, although seeds may be found occasionally.

Growth and ripening of the multiple fruit proceeds from the base to the

apex. The time between the ripening of the fruitlets at the base and those at the apex may extend over a period of from one to four weeks. The development of the various physicochemical properties of the fruit, such as acidity, tissue translucence, accumulation of sucrose, etc., proceeds also from base to apex, i.e., in the same order as growth and ripening. This condition complicates considerably the selection of fruits of identical physiological and chronological growth.

EXPERIMENTAL

Four different sets of fruits grown during different years and under slightly varying field and climatic conditions were studied. The total number of fruits in each set, ranging from 100 to 200 individuals, were selected for uniformity of size and stage of development at the time of red bud emergence. Of these selected fruits, 5, 15, 10 and 10 fruits, respectively, were taken from the four sets, at each growth period for weighing, etc. In most such fruits the basal flowers started to open about two or three weeks after the time of selection. The fruits were picked at two-week intervals and brought to the laboratory where their volume was determined by water displacement and weights were obtained of the whole fruit and its sections. In certain cases, the total number of fruitlets of the entire fruit was computed to obtain mean weight values of the fruitlets at different growth stages.

The determinations for two of the sets mentioned above were made in 1933 (March to July), and for the remaining two in 1935 and in 1937. The results obtained in 1933 are reported in tables 1 to 6 and figures 3 to 6. The fruitlet weight values were obtained from the 1937 set of fruits and are reported in Table 8 and Figure 8.

In order to study the type of growth of the fruits and of the sections, namely, flesh, shell and core, made at different chronological periods, the data were reduced to fit the monomolecular equation of autocatalytic reactions as employed by Robertson (5).

$$\log \frac{x}{A - x} = K(t - t_1) \qquad \text{(I)}$$

where $K = K_1 A$, and t_1 is the time at which the reaction is half completed, that is, when $x = (1/2)A$. The values of $K(t - t_1)$ were obtained from Robertson's Table LX on page 329. The same experimental data were also calculated with the equation as modified by Bray and Davis (2)

$$\log \frac{x + a}{A - x} = \log \frac{a}{A} + Kt \qquad \text{(II)}$$

where A = the final weight of fruits, etc., and X = the weight at different growth stages. The difference between the two equations is in the value of the initial weight of the fruit or plant at zero time. This value, designated by a, is very small in comparison to x, and almost negligible in the later stages of fruit or plant growth. However, in the early stages of growth, its significance should

not be neglected because it constitutes a large percentage of the total weight of the fruit or plant.

Both Robertson (5) and Reed (4) found, with Equation (I), considerably lower observed than calculated values of x in the early stages of growth. Reed (4) by employing Equation (II) was able to reduce considerably the divergence or "lag period" between the observed and calculated values of x during the early growth stages of *Scenedesmus acutus* colonies.

The importance of taking into account the value of a is brought out very clearly by Bray and Davis's statement that a "pure" autocatalytic reaction will never start unless x_0 has a finite value.

Equation (I), in its application to biological growth reactions, has been amply discussed by Robertson (5) and was also briefly presented in a former publication by the authors (4). Equation (II) has been discussed by Bray and Davis (2) in its application to the reactions involved in the autocatalytic reduction of bromate by hydrogen peroxide in acid solutions. Reed (4) was the first to demonstrate the application of Equation (II) to the growth phenomena of colonies of *Scenedesmus acutus*. The latter author has also presented a simplified procedure for solving and applying this equation to biological data.

RESULTS

The observed and calculated values obtained with either Equation (I) or (II), for two different sets of fruits, may be compared in tables 1 and 2. They

TABLE 1. GROWTH OF PINEAPPLE FRUITS (WEIGHTS) OF CLONE 54 CALCULATED FROM EQUATION (II): $\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt$; WHERE $a=31$, $K=0.0226$ AND $A=2750$; AND FROM EQUATION (I): $\log \frac{x}{2750-x} = 0.0221(t-86)$

t Days	x Obs. gm.	Equation (II)			Equation (I)	
		$Kt+\log \frac{a}{A}$	x Calc. $= \frac{Am-a}{1+m}$	Diverg. %	x Calc.	Diverg. %
0	—	−1.948	—	—	—	—
14	31	−1.632	31	0	67	116.0
28	100	−1.316	96	4.0	137	37.0
42	236	−1.000	222	5.9	268	13.5
56	438	−0.683	448	2.3	490	12.0
70	792	−0.368	802	1.3	844	6.5
84	1325	−0.048	1282	3.2	1305	1.5
98	1674	+0.264	1773	5.9	1780	6.3
112	1984	0.582	2178	9.8	2195	10.5
126	2394	0.902	2440	1.9	2432	1.6
140	2565	1.212	2585	0.8	2580	0.6
154	2725	1.532	2670	2.0	2670	1.7
168	2750	1.852	2712	1.4	2710	1.5

show that the divergence between observed and calculated values obtaining during the early stages of growth was considerably smaller with Equation (II)

than with Equation (I). That Equation (II) gave a better fit between observed and calculated values of X is shown below as obtained by the standard error of estimate for the curvilinear function by equation

$$S^2y \cdot f(x) = \frac{(z'')^2}{n-1}$$

according to Ezekiel (3, p. 115). The standard error of estimate serves to indicate the closeness with which new estimated values may be expected to

TABLE 2. GROWTH OF PINEAPPLE FRUITS (WEIGHTS) OF CLONE 52 CALCULATED FROM EQUATION (II): $\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt$; WHERE $a=37$, $K=0.0211$ AND $A=2415$; AND FROM EQUATION (I): $\log \frac{x}{2415-x} = 0.0227(t-84)$

t Days	x Obs. gm.	Equation (II)			Equation (I)	
		$Kt+\log \frac{a}{A}$	x Calc. $=\frac{Am-a}{1+m}$	Diverg. %	x Calc.	Diverg. %
0	—	−1.815	—	—	—	—
14	37	−1.520	37	0	60	62.0
28	77	−1.225	101	31.0	122	58.0
42	222	−0.929	221	0.5	242	9.0
56	417	−0.635	425	1.9	453	8.6
70	787	−0.335	740	6.0	787	0
84	1208	−0.043	1122	7.1	1208	0
98	1544	+0.255	1542	1.3	1630	5.5
112	1806	0.555	1884	4.3	1960	8.5
126	2228	0.845	2128	4.5	2175	2.4
140	2295	1.135	2252	1.9	2295	0
154	2415	1.435	2338	3.2	2355	2.5

approximate the true but unknown values. The results obtained for the growth Equations (I) and (II) in clones 52 and 54 were as follows:

	Equations:	(*I*)	(*II*)
Clone 52:	$Sy \cdot f(x) =$	26.1	10.0
Clone 54:	$Sy \cdot f(x) =$	38.8	4.2

The above values show quite conclusively that Equation (II), on account of its smaller values and consequently its smaller degree of divergence, is a much better equation to employ in studies of this kind. Equation (II) was also used to calculate the values for x of the fruitlets and the flesh, shell, and core sections of the fruit.

Examination of the curves in figures 2 to 7 shows that there is a very satisfactory agreement between observed and calculated values of x. The percentages of divergence between observed and calculated values of x is comparatively small and within the range of variability of the experimental material.

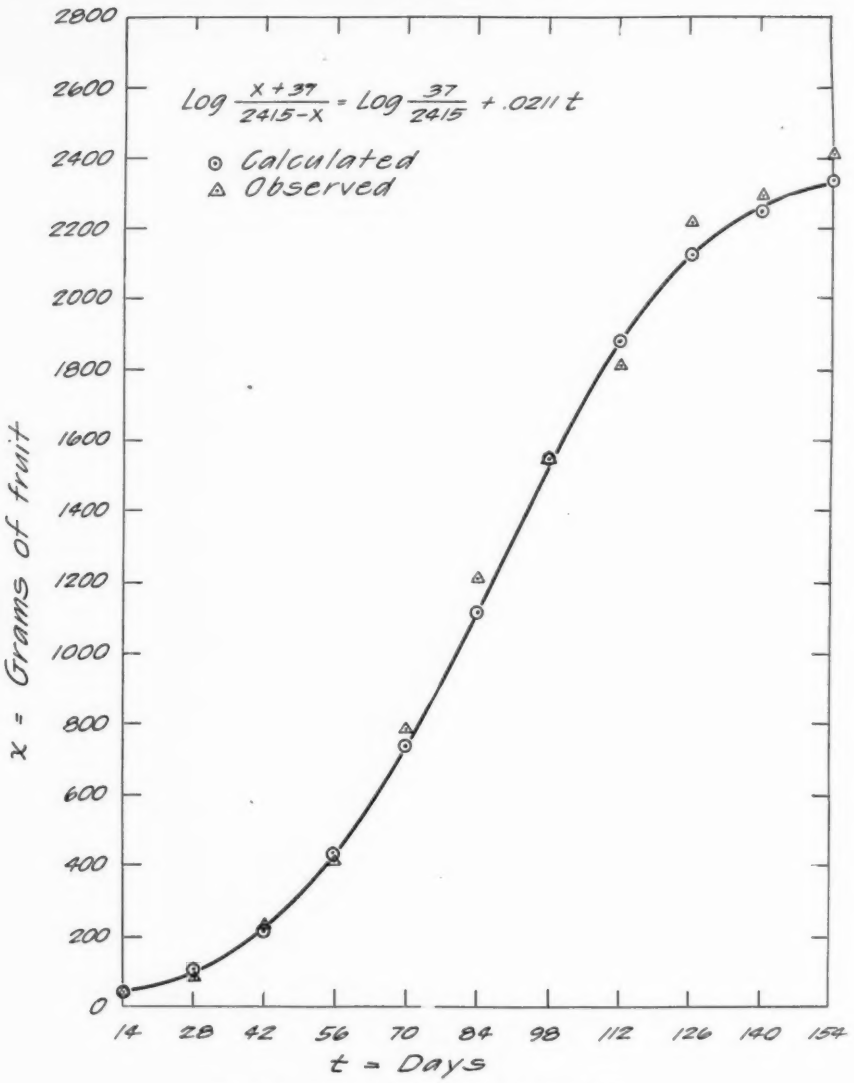

FIG. 2. Growth (weights) of (whole) pineapple fruits.

Fruit Weights.—The rate of growth (weights) of the fruits of all three sets increased until about the end of the 85th day period as shown in Table 3. After that time it decreased gradually. However, at the end of the 126th day period the amounts of tissues produced in the sets of clones 52 and 54 increased unexpectedly. This condition could not be explained and in 1937 additional data

TABLE 3. WEIGHT OF FRUIT TISSUE INCREMENTS PRODUCED BY CLONE 54 AND 52 AND SET OF 1937 AT DIFFERENT INTERVALS

Time interval days	Clone 54 gm	Clone 52 gm	Set of 1937 gm
14– 28	69	40	74
29– 42	136	145	126
43– 56	202	195	147
57– 70	354	370	290
71– 84	533	421	449
85– 98	349	336	585
99–112	310	262	317
113–126	410	422	300
127–140	171	67	179
141–154	160	120	—

TABLE 4. GROWTH OF PINEAPPLE FRUIT FLESH (WEIGHTS) OF CLONE 52 CALCULATED FROM

$$\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt;$$ WHERE $a=9$, $K=0.0246$ AND $A=1776$

t Days	x Obs. gm	$Kt+\log \frac{a}{A}$	x Calc. $=\frac{Am-a}{1+m}$	Diverg. %
0	—	−2.295	—	—
14	9	−1.950	9.9	10.0
28	22	−1.605	34.0	55.0
42	88	−1.263	85.0	3.4
56	196	−0.915	186.0	5.1
70	466	−0.573	368.0	21.0
84	784	−0.227	656.0	16.3
98	1050	+0.115	1003.0	4.5
112	1192	0.465	1320.0	10.7
126	1602	0.805	1536.0	4.1
140	1662	1.145	1660.0	0.1
154	1776	1.495	1725.0	2.9

TABLE 5. GROWTH OF PINEAPPLE FRUIT SHELLS (WEIGHTS) OF CLONE 52 CALCULATED FROM

$$\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt;$$ WHERE $a=24$, $K=0.020$ AND $A=522$

t Days	x Obs. gm	$Kt+\log \frac{a}{A}$	x Calc. $=\frac{Am-a}{1+m}$	Diverg. %
0	—	−1.338	—	—
14	24	−1.058	25	4.1
28	45	−0.778	54	20.0
42	109	−0.498	108	0.8
56	170	−0.218	182	7.0
70	250	+0.062	268	7.2
84	351	0.342	353	0.6
98	391	0.622	417	6.6
112	494	0.902	462	6.5
126	522	1.182	488	6.5
140	522			
154	522			

were obtained on a set of 15 fruits. The data in the fourth column of Table 3 show that this unexpected rise in the values for the former two sets of fruits

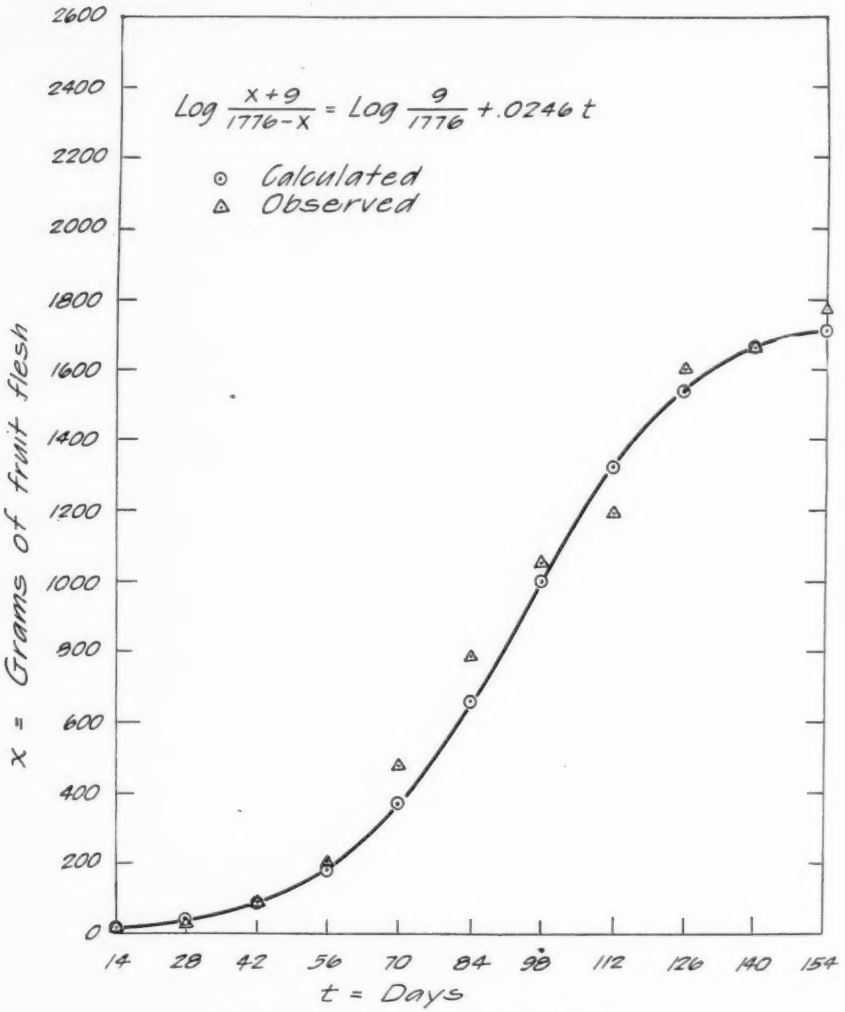

FIG. 3. Growth of the flesh tissue of pineapple fruits.

did not appear for those of the 1937 fruit. It is possible that the increase in weights for this period was due to the selection in sampling of fruits considerably larger than those of the average weight of the general fruit population or changes in the environment about which we have no information.

Weights of Flesh, Shell and Core.—The weights of the flesh, shell and core sections presented in tables 4 to 6 show a somewhat greater percentage diver-

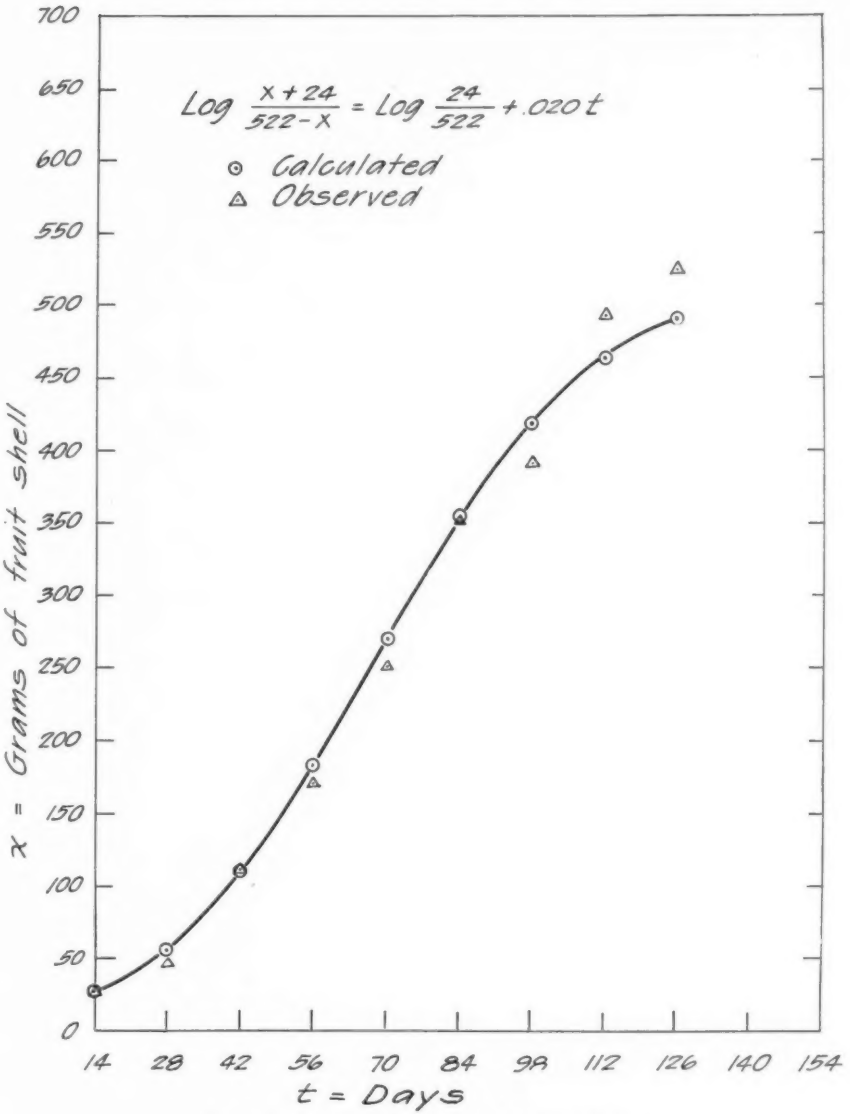

FIG. 4. Growth of the shell tissues of pineapple fruits.

gence between observed and calculated values than was found in the values for whole fruits. This may have resulted through slight losses of the sap during dissection.

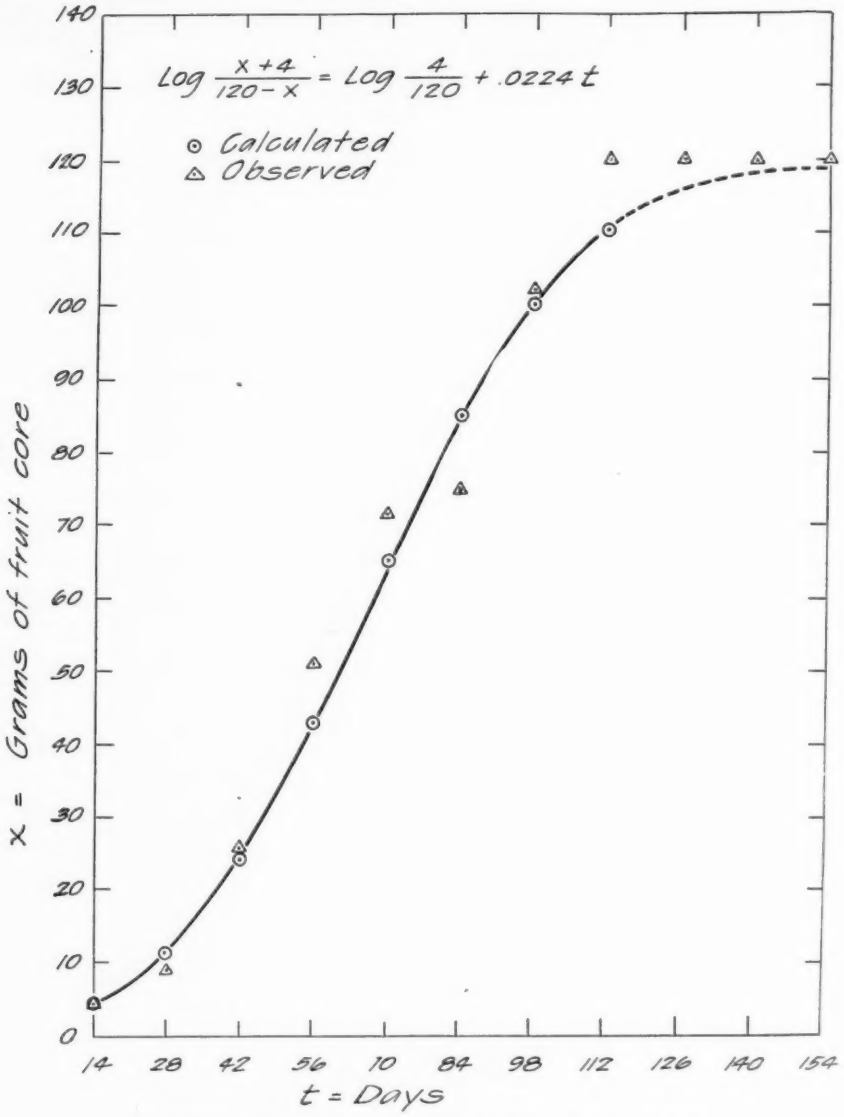

FIG. 5. Growth of the core or central cylinder tissues of pineapple fruits.

The data show that the time for completion of growth was different for the three different fruit sections. The core completed its growth at the end of about the 112th day period; the shell at the end of about the 126th day period and the flesh continued to increase in weight till ripening and harvesting, i.e.,

TABLE 6. GROWTH OF PINEAPPLE FRUIT CORES (WEIGHTS) OF CLONE 52 CALCULATED FROM

$$\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt;$$ WHERE $a=4$, $K=0.0224$ AND $A=120$

t Days	x Obs. gm	$Kt+\log \frac{a}{A}$	x Calc. $=\frac{Am-a}{1+m}$	Diverg. %
0	—	−1.477	—	—
14	4	−1.163	4	0
28	9	−0.849	11	22
42	25	−0.535	24	4
56	51	−0.222	43	16
70	71	+0.093	65	8
84	74	0.403	85	15
98	103	0.718	100	3
112	120	1.033	110	8
126	120			
140	120			
154	120			

TABLE 7. SPECIFIC DENSITY $\left(\frac{\text{WEIGHT}}{\text{VOLUME}}\right)$ OF PINEAPLE FRUITS AT DIFFERENT STAGES OF GROWTH AND DEVELOPMENT

Stages of Growth Days	Specific Density
14	0.882
28	0.946
42	0.910
56	0.942
70	0.935
84	0.974
98	0.970
112	0.990
126	1.045
140	1.060
154	1.075
168	1.114

TABLE 8. GROWTH OF PINEAPPLE FRUITLETS (WEIGHTS) ON PLANTS WITH ONE SLIP CALCULATED FROM

$$\log \frac{x+a}{A-x} = \log \frac{a}{A} + Kt;$$ WHERE $a=0.482$, $K=0.0188$ AND $A=15.600$

t Days	x Obs. gm	$Kt+\log \frac{a}{A}$	x Calc. $=\frac{Am-a}{1+m}$	Diverg. %
0	—	−1.512	—	—
14	0.482	1.249	0.372	22.8
28	1.065	0.986	1.018	4.4
42	2.130	0.722	2.085	2.1
56	3.740	0.458	3.660	2.1
70	6.100	−0.196	5.785	5.2
84	8.170	+0.068	8.190	0.2
98	10.300	0.330	10.460	1.5
112	11.460	0.592	12.320	7.5
126	13.710	0.858	13.650	0.4
140	15.600	1.118	14.500	7.0

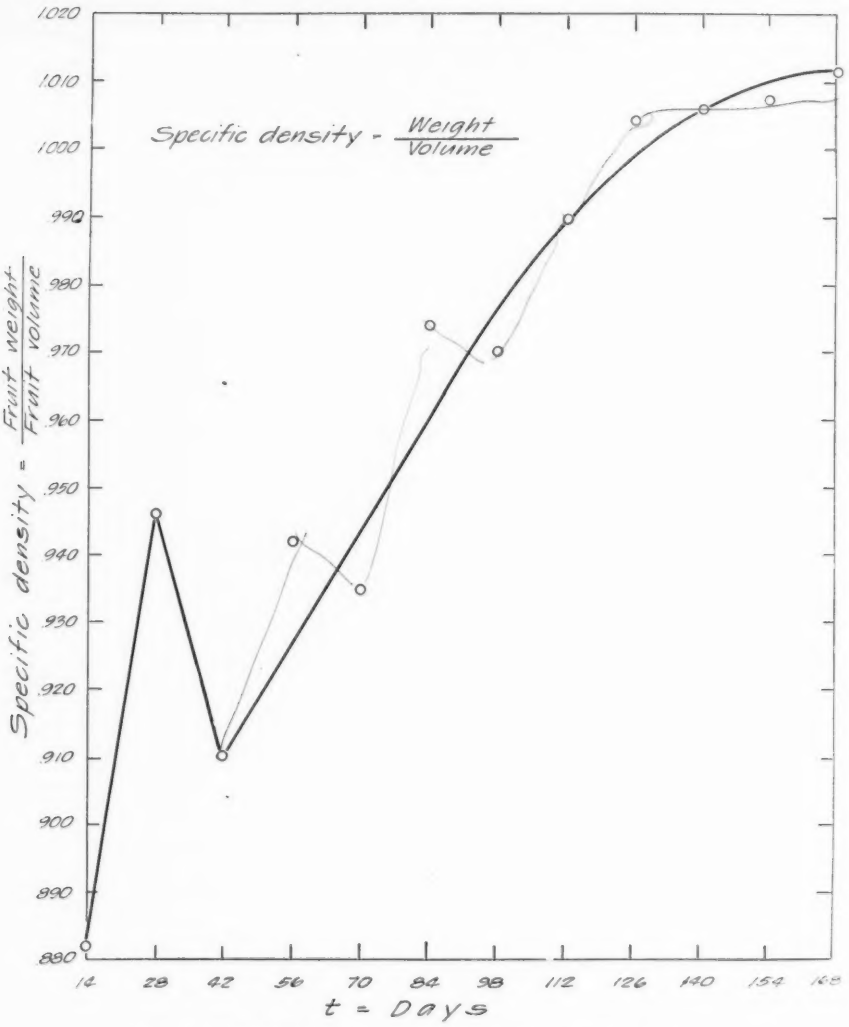

FIG. 6. Specific density of pineapple fruits at different growth stages.

until the end of the 154th day period. The interval between the end of the 112th and 154th day periods may be called the maturation period of the fruit. During this period, the flesh and shell tissue of the apical portion of the fruit continued to grow, while the tissues that had completed growth began and continued to accumulate sucrose and organic acids, according to certain unpublished data, and to undergo also various other metabolic, physicochemical and morphological changes. The increases in weight of flesh tissues after the

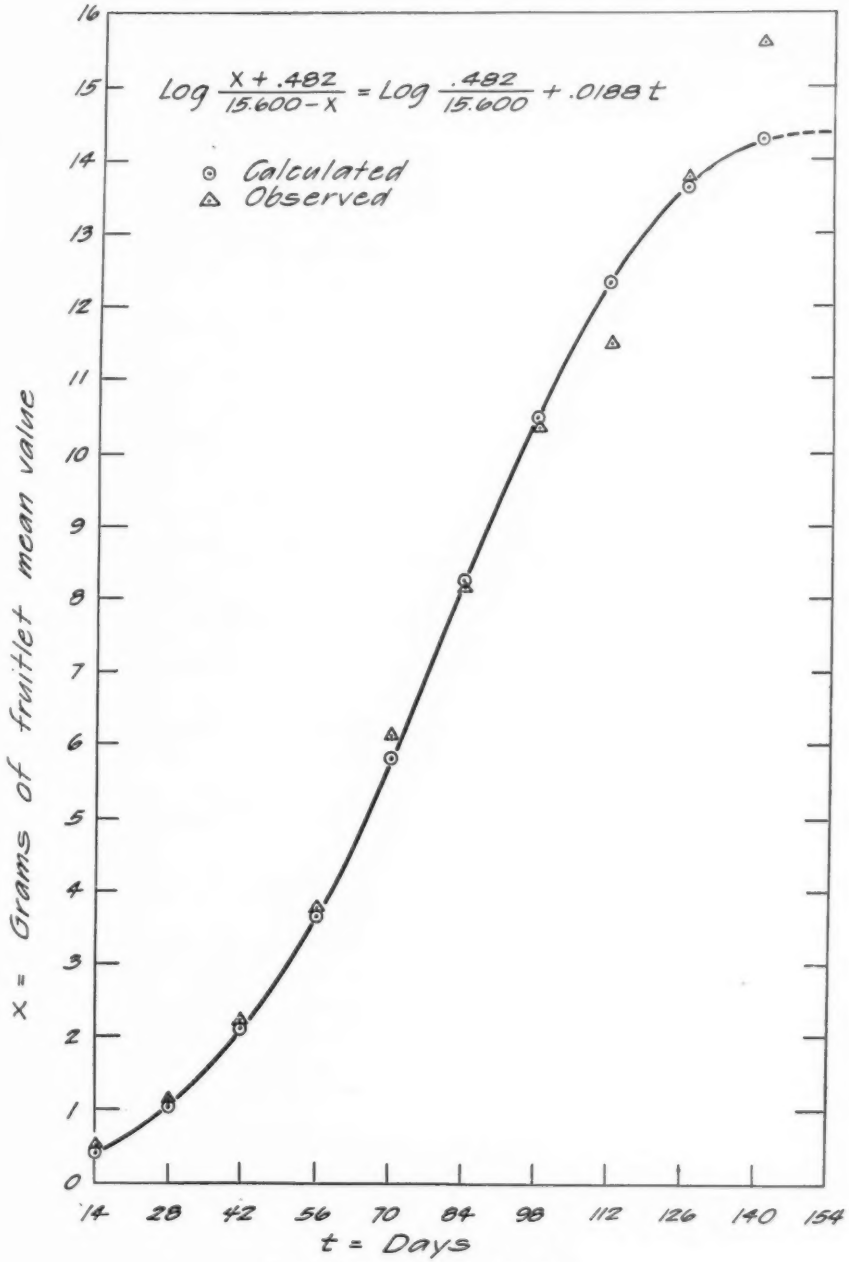

FIG. 7. Growth of pineapple fruitlets.

end of the 126th day period were due to greater specific density values of the fruits as shown in Table 7 and Figure 6.

Fruitlet Weights.—The fruits which were selected for the study of fruitlet weights had an average number of 144 fruitlets. The weight of the individual fruitlets varied; those at the region of the base weighed more than those at the apex. A mean fruitlet weight per fruit was obtained by dividing the weight of the entire fruit by the number of its fruitlets.

The growth curve for fruitlets, based on periodic mean weight values, is shown in Table 8 and Figure 7. The percent divergence between observed and calculated values was, with the exception of the first and last periods, comparatively small.

DISCUSSION

The observed values of the weights of whole fruits and fruitlets were lower than the calculated values at the end of the 112th day period, according to the data presented in tables 1-B, 2-B and 8 and figures 2 and 7. The divergence may be due either to lack of uniformity in sampling, or to a partial inhibition of the growth of the fruit resulting from the rapid development of slips. Slips are vegetative proliferations produced at the apical end of vestigial fruits known also as slip balls which are borne laterally on the peduncle of the fruit proper, as shown in Figure 1. The weight of slip balls varies greatly as well as the number of their component fruitlets. The time required for the growth of the slip ball is more or less directly proportional to the number of its fruitlets as is also true with the fruit. However, as the number of slips per plant may vary from

TABLE 9. FRUIT WEIGHTS (GRAMS) OF TWO STRAINS OF PINEAPPLE PLANTS; A, "FEW-SLIPS" AND, B, "COLLAR-OF-SLIPS" OF CLONE 54 AT DIFFERENT GROWTH INTERVALS

Growth Interval Days	Strains	
	Few Slips gm	Collar of Slips gm
14	31	—
28	102	—
42	238	—
56	457	361
70	856	739
84	1415	1162
98	1806	1357
112	2113	1520
126	2665	1958
140	2558	1990
154	2750	2462

0 to 6 and in strains with many slips, known also as "collar of slips," this number may be considerably greater, the growth inhibiting effects of slips on fruit weights vary considerably. Such effects may be observed in Table 9 where the weights of the fruits of plants from two strains of the same clone are reported. The data show that the weights of the fruits of the "collar-of-slip"

strain were considerably lower than those of the "few-slip" or desirable strain. Unpublished data of J. L. Collins show that numbers of slips and fruit weights are inversely related. Data on the average weights of slips obtained at different growth periods were as follows:

Growth Period Days	*Av. wt. per Slip* Grams
56	9
70	22
84	40
98	49
112	76
126	90
140	110

When the effects of the variability of the experimental material and of fruit growth inhibition, resulting from the growth of slips, are taken into consideration, the divergence between observed and calculated values are within the range of the experimental error and the agreement, therefore, between observed and calculated values quite satisfactory. With all the above conditions in mind, it is safe to state that the curves obtained from reactions involved in the growth phenomena of pineapple fruits conform very closely to those of autocatalytic monomolecular reactions.

SUMMARY

The growth curve of pineapple fruits conforms very closely to that of autocatalytic monomolecular reactions. The experimental data were reduced to fit the equation as recommended by Robertson where:

$$\log \frac{x}{A - x} = K(t - t_1) \qquad \text{(I)}$$

and also a modified form of the same equation as recommended by Bray and Davis where:

$$\log \frac{x + a}{A - x} = \log \frac{a}{A} + Kt. \qquad \text{(II)}$$

The agreement between observed and calculated values of x was found to be better, particularly at the earliest growth stages, when Equation (II) was employed. In Equation (II) the initial weight of the fruits (a) was taken into account when x was equal to zero time, whereas in Equation (I) this item was entirely left out.

The percentage divergence between observed and calculated values has been attributed in part to a lack of absolute uniformity of the experimental material and also to the inhibiting effects of slips on fruit growth.

Data obtained for various sections of dissected fruits showed that the core reached full development before the shell. The flesh of the fruit was the last to

reach complete growth and development. The interval between complete development of core and fruit ripening has been designated as "maturation period," because the tissues during this time attain maturity and accumulate, according to other unpublished data, sucrose, organic acids, carotenoid pigments, esters, etc.

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BASING WEIGHT STANDARDS UPON LINEAR BODILY DIMENSIONS

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In this paper we shall discuss three subjects which—if we can judge by the published material—have long been of interest to students of the physical status of individuals. These subjects are: (1) types of body build; (2) indices describing body build; (3) the relation of certain physical measurements to body weight. The first of these, we shall (partly in this paper, and more fully in another) attempt to show, is a relic of a psychological error carried over to the study of physical anthropology; we shall question the utility of the second; and the third will be discussed with special reference to the undesirability of using single measures, and therefore the necessity of obtaining combined measures for the determination of normal body weight.

The greatest stimulus to recent studies of body types was undoubtedly that of Kretschmer[10] whose findings concerning the relationships between body types and behavior types are too well known to need complete description here. Kretschmer found that there was a close relationship between his pyknic physical type (tendency to be fat, round) and the cycloid disease type, while asthenic (tall, slender) and athletic (strong, broad, muscled) types suffered from diseases of the schizoid type. These findings reported by Kretschmer have stimulated a great deal of interest in body types notwithstanding the fact that his physical type groupings were not made by means of carefully obtained and accurately combined measurements, and no information is given concerning the previous physical and disease history of his subjects.

Since the limitations of Kretschmer's study were not readily ascertained, his findings stimulated a series of studies concerning body types. The relation of body types to other characteristics in the studies which have been conducted (Klineberg, Asch, and Block,[9] Bayley,[1] Heidbreder,[5] Sheldon[14]) have always been low. We shall offer later in this paper a possible explanation for such findings.

The study of body types has led further to the development of numerous indices for describing body build. Jackson[7] states the relationship as follows:

> The body build or habitus indicates in general the relative thickness of the body in proportion to its length. It is measured *most conveniently* by an index of body build, expressed as a ratio between weight and height. For this purpose the square or the cube of the height is generally *preferred*.

It appears to the present writers that the two words "conveniently" and "preferred" express the points to which they make the strongest objection. For the purposes of "convenience" and on the basis of "preference" we have

indices computed, presented and utilized for such purposes as studying body build in relation to number of suicides, ability to earn college grades, body weight, and general intelligence.

The results obtained from experiments such as those noted above, and an inquiry into the logic of the whole problem of body build leads us to doubt the value of much of the previous work. It would be possible to get exactly the same index for a short, fat, wide and deep body as for a long, thin, narrow and shallow body by using the ratio between height and weight and, having obtained the equal indices, little is known about the individuals who are described by them. Suppose one individual to be five feet tall and to weigh one hundred pounds. He would have the same index (Ht. 2/Wt.) as an individual six feet tall weighing one hundred forty-four pounds.* The index, like the IQ, gives a relative measure, and possesses all the disadvantage of such measures. Even when a measure other than weight is used, we still have an index which is of doubtful value. Lucas and Fryor[11] have, for example, used an index of height divided by iliac width. We shall show in another paper that a child may have a series of indices from ages 7 to 17 which show variation only in the third decimal place over that age period, while the individual has gained the difference in weight between a child of 7 and 17.

We must then, in view of the above, question the value of using the common indices as measures of anything in particular. But even if some value can be found for the common indices, we must question their general use for two other reasons. The first of these is the fact that there has been no determination of the extent to which the items in the indices should be "weighted" before being combined; that is, the assumption that there is a one-to-one or one-to-one squared, or one-to-one cubed relationship may or may not be valid. The second is that few of the indices give due consideration to the fact that measures of breadth and depth are as closely related to weight as is height. The use of a single measure of height as an index to describe individuals who may be as much as five sigmas of the distribution of measures of width and depth apart does not seem justified.

It is this last factor which leads us to question the value of height-weight norms as such. Jackson[7] reports correlations of only .48 between height and weight for a million draft recruits, of only .52 for a sample of 80,000 recruits at demobilization, and .50 for the relation between heights and weights of the student group at the University of Minnesota. We have found higher coefficients (.68) for 533 sixteen-year-old boys measured by the Harvard Growth Study. It must be remembered, however, that even the largest of these correlations means that we could place less than 22 percent of our subjects within half a point of the position predicted on a ten-point scale.[6] Such findings lead

* Study of indices derived from height and weight indicates that the two measures are used as if they were independent of each other. Weight and height are not unrelated, as coefficients of correlations between the variables show. The use of such indices then involves dividing one number by another, the second being a function of the first. It is difficult to say just what the result is and more difficult to evaluate it.

us to consider the inadequacy of tables based merely upon the relationship between height and weight.

The discussions presented in brief summary above have led us to investigate the questions of body build, indices of body build, and the relationship between body build and body weight. For this purpose we have used as subjects a group of cases selected from the files of the Harvard Growth Study.[2]

The first step in our work consisted of examining the relationship between body weight and a sampling of indices of body build which have been suggested in the literature.[11] We selected the measurements of a group of sixteen-year-old boys at random from the files of the Crowth Study, computed the various indices and plotted the indices against the measurements of weight. The results are presented in Charts I, II, III, IV, V. It should be pointed out that these charts are not intended to be correlation charts but are simply used to indicate the general trend of the relationships found.

Examination of Chart I reveals that the relationship between the Lucas and Pryor index (iliac ÷ height) and weight is not close enough that accurate estimate of weight can be made upon the information presented by the index. It can be seen, for example, that boys of the same age with the same index of body build (.169) may vary in weight from 45 to 78 kilos (66 pounds), while individuals of the same weight—60 kilos—may vary in index from .151 to .178. The distribution in the chart indicates that the Lucas and Pryor index is not a close correlate of body weight.* Its advantage in predicting normal weight would seem to be less useful than knowledge of height alone as is shown in Chart II, in which we have plotted height against weight for the same individuals.

Tilton's index[15] which was worked out by means of the water displacement method is not a good indicator of body weight, as is shown in Chart III. It appears to be better than the Lucas and Pryor index but the small gain can hardly be worth the extra labor required in its computation.

An index that we had tried before deciding upon the procedure outlined later in this report was obtained by multiplying height by chest depth by iliac in order to get a cubic measure is plotted against weight in Chart IV. The index numbers are unwieldy but they show a closer relationship between weight than do the others mentioned, and closer than another index (Chest Depth × Iliac/Height) which is plotted against weight in Chart V.†

The results of our preliminary examination, then, reveal that the indices considered here are not closely related to body weight. The reason for the lack of relationship appears to be due to either of two factors, namely, the fact that the measurements have not been "weighted" with due consideration to their relationship to body-weight,‡ and secondly to what appears to be a mistaken

* We shall show in another study that the Lucas and Pryor index is very consistent for the same individuals over an eleven-year period. It does not, however, appear to be closely correlated with weight at any level.

† The same subjects were used in plotting Charts I to V with a few exceptions where the measurements for some subjects were incomplete.

‡ "When measures are not weighted, they weight themselves." Hull, C. L. *Aptitude Testing*, World Book Co., 1930, p. 457.

CHART I

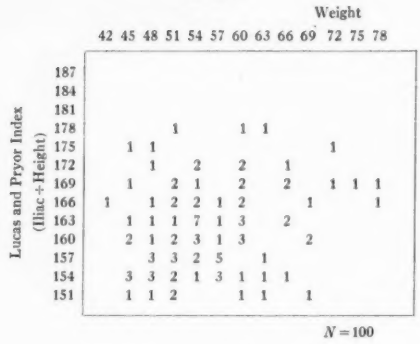


CHART II

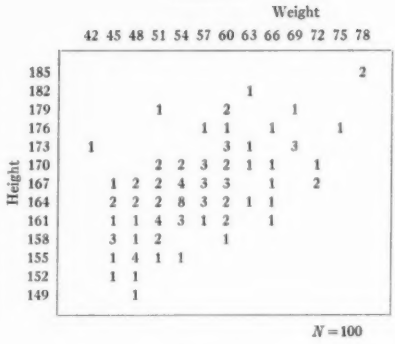


CHART III

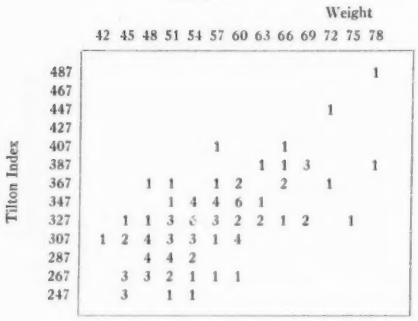


CHART IV

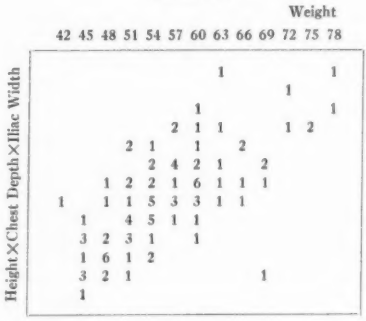
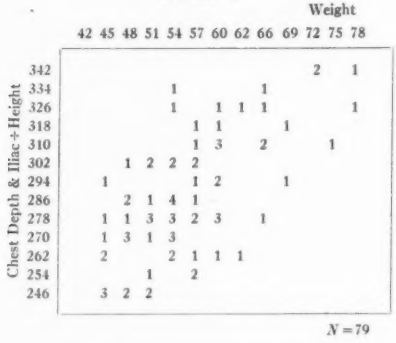


CHART V



idea—that individuals can be classified into body types on the basis of one or two measurements and that the index of body type so obtained will be closely related to body weight.

Now the classification of persons into types of body build presents a number

of problems, the analysis of which throw considerable doubt on the value of such classifications as those of Kretschmer[10] and others. The difficulty lies in determining just when one type ends and another begins. If broad two-class categories are used, we can select from the Harvard Growth Study, individuals possessing any grouping (not counting the mutually exclusive categories) of the following characteristics:

Tall	Short
Long trunk	Short trunk
Long legs	Short legs
Wide chest	Narrow chest
Deep chest	Shallow chest
Wide iliac	Narrow iliac
Heavy weight	Light weight

One individual might be found* who possessed all the characteristics mentioned in the left column, another, with all those in the right column, and numerous individuals can be found with various combinations from each of the columns. Just using the two classifications listed we might find individuals with the following characteristics. These are, however, only samples from the hundreds of groupings that could be found.

Individual A	*Individual B*	*Individual C*	*Individual D*
Tall	Opposite of A	Tall	Short
Long trunk	in all respects	Short trunk	Short trunk
Long legs		Long legs	Short legs
Wide chest		Wide chest	Narrow chest
Deep chest		Shallow chest	Shallow chest
Wide iliac		Wide iliac	Narrow iliac
Heavy weight		Heavy weight	Light weight

But if the number of combinations seems impressive when only two categories are used, consider the problems when we try to define the terms, tall, shallow, deep, light-weight, and when we attempt to put these into numerical terms. Consider that even when we try to use three categories, we get the classes listed below with all the possible combinations of them.

Tall	Average	Short
Long trunk	Average trunk	Short trunk
Long legs	Average legs	Short legs
Wide chest	Average chest width	Narrow chest width
Deep chest	Average chest depth	Shallow chest depth
Wide iliac	Average iliac	Narrow iliac
Heavy weight	Average weight	Light weight

And if we use 1 sigma as a measure, consider the number of possible combinations:

* See Table 1.

-3σ	-2σ	-1σ	$+1\sigma$	$+2\sigma$	$+3\sigma$
Height	Height	Height	Height	Height	Height
Legs	Legs	Legs	Legs	Legs	Legs
Trunk	Trunk	Trunk	Trunk	Trunk	Trunk
Weight	Weight	Weight	Weight	Weight	Weight
Chest Width	Chest Width	Chest Width	Chest Width	Chest Width	Chest Width
Chest Depth	Chest Depth	Chest Depth	Chest Depth	Chest Depth	Chest Depth
Iliac	Iliac	Iliac	Iliac	Iliac	Iliac

The illustrations given above are only suggestive of the thousands of types of body build which are theoretically possible. It appeared to us that we might examine to what extent such types would appear in the measurements of a group of 533 sixteen-year-old boys selected at random from the Growth Study files.* It also appeared that in classifying these individuals into types, the only way to make such satisfactory classifications was in terms of the extent to which individuals vary from the average—that is, a boy can be considered tall, or wide-chested, or of light weight only with respect to the extent of his variation from the average measurements of groups of his own age with which he might be compared.

The procedure for determining and describing individual variation from the average of a group in terms of comparable scores has been outlined by Kelley[8] and we have adopted for our purposes his standard score technique. For each of 7 measurements we have obtained the mean and sigma for sixteen-year-old boys. We have then computed the standard score for each measurement and, in order to save working with decimals, we have rounded off the standard score to two places and multiplied by 10. Thus a score of -37 in standing height means that the individual in question is approximately 3.7 sigmas below the mean of the heights of sixteen-year-old boys. A score of $+10$ would indicate that the individual is 1 sigma above the mean of the height measurements of the same group.

With this procedure in mind we may turn to Table 1 and examine the deviations that may occur within one individual with respect to the average of his group. Individual *A* is consistently far *below* average in all measurements and *B* is consistently far *above* average in all measurements. *C* is of average height with very slightly shorter than average legs but with a slightly longer than average trunk. His iliac and chest measurements are above average, as is also his weight. Individuals *A*, *B*, and *C* are the kind that would be included in types and described as (A) short, narrow and shallow, (B) tall, broad and deep, and (C) as average. It must be noted, however, that, even in these cases which would be classified as types, there is considerable variation in the extent to which they deviate from average in each of the measurements.

Individual *D* is below average in height, trunk, and legs, and one might expect therefore that he would be of light weight, but we find that he is above

* Ethnic origin of groups: 68% Northern European, 21% Italian, 6% Jewish, 4% Southern European, and 1% Negroes.

average weight. The latter condition is probably due to the fact that his iliac and chest width measurements are well above average. His chest is of average depth. *D* might be described, then, as short, broad and shallow. *E* is like *D*.

F is of average height with short legs and long trunk, but his iliac is broad, and he is slightly above average in chest measurement. The greater than average width and depth measurements account for his more than average weight.

G is taller than average, has long legs and short trunk with nearly average chest and iliac measurements. His weight is significantly above average. *G*'s measurements suggest that very long legs, even combined with a short trunk, make for greater than average weight.

Individual *H* is short and light; he has a short trunk and legs, a narrow and shallow chest, but his iliac is above average width. His weight is not so far

TABLE 1. SAMPLES* OF INDIVIDUAL DEVIATION IN BODILY CHARACTERISTICS FROM THE MEAN OF A GROUP OF 533 SIXTEEN-YEAR-OLD BOYS. FOR INTERPRETATION OF NUMBERS, SEE PRECEDING TEXT. DATA FROM THE HARVARD GROWTH STUDY

Individual	Standing Height	Weight	Leg Length	Trunk	Iliac	Chest Depth	Chest Width
A	−37	−37	−23	−40	−39	−29	−35
B	+28	+25	+22	+24	+29	+18	+25
C	0	+ 5	− 2	+ 1	+ 2	+ 4	+ 2
D	− 5	+10	− 6	− 3	+ 8	+ 1	+22
E	−12	+ 3	−14	− 7	+ 9	+ 1	+ 4
F	+ 1	+13	−10	+16	+28	+ 5	+ 6
G	+ 9	+ 8	+26	−15	+ 2	+ 4	− 5
H	−15	− 7	−18	− 3	+ 7	− 8	+ 3

* Complete standard scores for each of eight measurements on 533 sixteen-year-old boys have been worked out and will be discussed in detail in another paper.

below average as would be expected in consideration of his height. This condition appears to be due to the above average width of the iliac.

The cases presented in Table 1 have been drawn from a population of 533 and have been presented here simply to show the extent to which deviation from the average in several measurements can occur within the same individual. The cases illustrate the futility of trying to describe any but the cases at the extremes of the distributions as types. The standard scores of the whole group of 533 indicate that there is no consistent tendency for individuals to be above or below average to the same extent in the seven measurements. The tall individual may be narrow and shallow, with short trunk and long legs, or wide and deep, with long trunk and short legs. Many other combinations of such measurements have been found.

The findings here suggest reasons why height-weight tables are unsatisfactory, why indices using only a few (unweighted) measurements are not closely correlated with weight, and why classification of individuals into body types is a procedure which fails to assist us in the determination of normal weight.†

† The study of the longitudinal data of the Harvard Growth Study revealed another difficulty with height-weight tables. At years 16 and 17 we found individuals who had not grown (greater than the limits of error of measurement) for a year. Yet, according to the tables, they should have made increases in weight.

In view of the above discussion, the problem of determining normal weight for individuals appears to be a difficult task. There is a technique, however, by which the determination of normal weight for body build can be made in which each measurement is weighted according to its relative importance in determining body-weight. That technique is the regression equation.

To set up our regression equation for the prediction of normal weight from body build, we have computed the intercorrelations between eight measurements for the population of 533 sixteen-year-old boys from which we drew to give our illustration in Table 1. The correlation coefficients are presented in Table 2 which follows. The coefficients are worthy of study, for examination of Table 2 reveals that standing height is not so closely related to weight as is

TABLE 2. CORRELATION COEFFICIENTS BETWEEN EIGHT PHYSICAL MEASUREMENTS, AND THE MEANS AND SIGMAS OF THE MEASUREMENTS. FIVE HUNDRED THIRTY-THREE BOYS, SIXTEEN YEARS OF AGE. DATA FROM THE HARVARD GROWTH STUDY

	Weight (0)	Chest Width (1)	Sitting Height (2)	Iliac (3)	Trunk Length (4)	Standing Height (5)	Chest Depth (6)	Leg Length (7)
(1) Chest Width	.7174							
(2) Sitting Height	.7008	.5272						
(3) Iliac	.7002	.5599	.6124					
(4) Trunk	.6846	.5198	.9280	.5915				
(5) Standing Height	.6756	.4838	.8913	.5914	.7270			
(6) Chest Depth	.6521	.4548	.4146	.4626	.4147	.4158		
(7) Leg Length	.4651	.3070	.4592	.4239	.3558	.8613	.3004	
	M	M	M	M	M	M	M	M
	57.92	25.36	86.93	27.08	56.28	167.70	18.31	80.86
	σ	σ	σ	σ	σ	σ	σ	σ
	8.72	1.39	4.04	1.27	3.01	7.48	1.46	4.60

implied in the height-weight tables. Four other measurements are as closely, or more closely, related to weight than standing height. The differences between the first five coefficients reported in Column 0 of Table 2 are small and of doubtful significance so that for prediction of weight (if a single measure is to be used) measurements of the chest, of sitting height, trunk, or iliac, might be used with as good results as when height is used. Leg length is far inferior to the other measurements, and as we shall show later can be neglected if standing height is used. The findings presented in Table 2 make one question whether the usual indices have any sound basis, and make one wonder whether the dividing and multiplying by leg length and standing height in constructing indices was continued only because it had been customary to do so.

Using the figures presented in Table 2, we have computed the multiple correlation between the best weighted combination of seven measurements and body-weight. The Multiple "R" (Doolittle method) with body weight, of the best weighted combination of the seven measurements, was .883. The weights obtained indicated that the measurements are related to body-weight in the following order:

Chest Width closely related
Chest Depth, next most closely related
Standing Height, next most closely related
Iliac, next most closely related
Trunk, next most closely related
Sitting Height, next most closely related
Leg Length, least closely related

(The above statements do not mean that trunk, sitting height, and leg length are unimportant in determining body-weight, but rather that they are unimportant when we include the other four measurements—that is, when we use the first four measurements, there is little need of using the last three.)

It is a fortunate circumstance that the four measurements which are most useful are most easy to obtain. Both trunk and leg length are derived measurements, and the securing of a measure of the trunk requires measurement of sternal height—a measure difficult to get on many chests. Sitting posture interferes with accurate measurement of sitting height.

Using the first four measurements, we have computed another multiple "R" between body-weight and the best weighted combination of chest depth, chest width, standing height and iliac. The multiple "R" here is .871—further evidence that we have obtained almost as much from four measurements as we had previously obtained from the seven. At the same time we have cut down the amount of labor and have avoided some measures which are difficult to obtain.

The weights obtained from our four measurements indicate that the order in which our measures are related to body-weight are the same as are listed above, namely: (1) Chest Depth, (2) Chest Width, (3) Iliac, and (4) Standing Height. From the weights we obtain the following regression equation for sixteen-year-old boys.

Weight (kilos) = 1.73 Chest Depth + 2.11 Chest Width + 0.28 Standing Height + 1.60 Iliac − 118.4

We may illustrate the use of the procedure by an actual case (#2900) chosen at random from the Harvard Growth Study. At age 16 this boy had the following measurements:

Chest Depth	16.4
Chest Width	23.6
Standing Height	176.9
Iliac Width	27.2

Multiplying through by the derived weights of the equation, we get:

$$\begin{aligned} 16.4 \times 1.73 &= 28.2 \\ 23.6 \times 2.11 &= 49.8 \\ 176.9 \times 0.28 &= 49.5 \\ 27.2 \times 1.60 &= 43.5 \\ \text{Total} &= 171.0 \end{aligned}$$

Subtracting correction factor for weighted averages, we find
expected (normal) weight from skeletal measures..52.6 kgs. (115.9 lbs.)
actual measured weight.....54.0 kgs. (119.0 lbs.)

difference (overweight).....+1.4 kgs. (3 lbs.)

The Baldwin-Wood Table for this boy gives weight of 145 pounds. He would ordinarily be considered to be 145–119 or 26 pounds under-weight.

We have put our formula to the acid test by trying it *on new groups of subjects*—boys who were not included in the original group on which the formula was devised, boys at ages other than sixteen, and girls aged fourteen to eighteen.

Now we cannot expect to get perfect prediction of body weight because we do not have measures of fat pads in buttocks, stomach, knees, hips, neck, (and even head!). We have no measures of unusual development of muscles on arms, legs, and neck. Nor have we considered the variance of a pound or two due to elimination, intake of food and drink, clothing, and some unreliability of measurement, so that when we get within ten pounds of actual weight we shall be doing very well indeed.*

The figures in Table 3 show that we have come very close to the actual weight in our prediction. With 50 sixteen-year-old boys the discrepancy between actual and predicted weight was *on the average* 5.5 pounds. (The height-weight table discrepancies averaged 11.4 pounds.) We were within 5 pounds of actual weight in 25 out of the 50 cases. In 17 of the 50 cases we were over 5, but less than 10, pounds off. Seven predictions were more than 10, but less than 15, pounds off, and in only one case were we more than 15 pounds (17 pounds) from the actual weight.

TABLE 3. COMPARATIVE PREDICTION OF NORMAL WEIGHT BY FORMULA AND HEIGHT-WEIGHT TABLES. FIFTY SIXTEEN-YEAR-OLD BOYS

	Formula	Height Weight Tables
Predicted within 5 pounds of actual weight	25	19
Predicted between 5 & 10 pounds from actual weight	17	11
Predicted between 10 & 15 pounds from actual weight	7	9
Predicted between 15 & 20 pounds from actual weight	1	0
Predicted between 20 & 25 pounds from actual weight		3
Predicted between 25 & 30 pounds from actual weight		5
Predicted between 30 & 35 pounds from actual weight		1
Predicted between 35 & 40 pounds from actual weight		2

The increase in value of our procedure over height-weight tables is seen in Table 3. We do six pounds better *on the average*, and we do not have such wide deviations as in the height-weight tables when we use the formula.

* There is some question concerning the logic of continuing beyond skeletal measures in predicting normal weight. We predict with our measures only what is to be expected in consideration of the measures of the skeleton. The variations from this weight must then be subject to special investigation—that is, we might then go on to measurement of fat pads (sub-cutaneous tissue) to determine whether or not they are the cause of the variance from normal weight. For discussions of the measurement of subcutaneous tissue see, Franzen, R.,[3] McCloy, C. H.,[13] Marshall, E. L.[12]

It appears then that we have been able to devise a rather good equation for the prediction of normal weight for sixteen-year-old boys. Our next problem was the determination of the extent to which the formula would be applicable to other than the sixteen-year level for boys, and to similar groups of girls.* All attempts at prediction of normal weight for groups of either sex below the fourteen-year level failed. The discrepancies obtained for fifty boys and fifty girls whose weight we attempted to predict at ages below the fourteen-year level were so large that there is no need of presenting them here. Apparently there is a change in relationships between measures of the body and body-weight in the distribution of measurements at adolescence. Such changes have been shown pictorially by Greulich.[4]

At age fourteen our predictions come within reasonable agreement with actual body weight. We have taken an entirely new population of fifty boys and fifty girls and have predicted the weights for each of our subjects at each age from fourteen to eighteen. The results are shown in Table 4 which follows. A study of the figures shows that for both sexes and at all ages from fourteen to seventeen, we have succeeded in predicting weight within 2.3 kilos (approximately 5 pounds) over thirty percent of the cases, and in from fifty-eight to seventy-four percent of the cases, we have succeeded in predicting within 4.6 kilos (10 pounds). In all ages from fourteen to seventeen, over eighty percent of the weights have been predicted with a discrepancy of less than fifteen pounds. The large percentage of cases (twenty percent) in which the discrepancies between actual and predicted weights are greater than fifteen pounds, is disconcerting, but, as we have stated previously, we do not have measures of fat and muscle pads, nor do we have any estimates of the influence of the effect of elimination, eating, and unusual glandular conditions. It should be pointed out that the main limitation of the regression equation is that it predicts more accurately at the mean than at the extremes, and it should also be noted that there is less need to resort to mathematics in the extreme cases, for wide variations from normal weight are readily observable.

In order to show the accuracy with which we have predicted body-weight by means of our formula, we have plotted the actual against the predicted weights for boys and girls at ages fourteen to eighteen in Charts VI and VII. The distributions in the charts presented contrast favorably with the distributions in Charts I to V and indicate rather clearly that our procedure is superior to others which have been proposed.† The evidence offered in these two charts in addition to that presented in Table 4 indicates that we have been successful in obtaining a formula by which normal weight can be predicted with slight

* It should be pointed out that in using the formula devised for sixteen-year-old *boys* at other age levels and for *girls* at several age levels, we should not expect to get accurate predictions of normal weight unless the variability within the groups and the size of the correlation coefficients are close to those obtained from our original group. If, however, the latter conditions are met approximately, our formula will be of more general use and will save working out a separate formula for each age and sex group.

† It must be noted that there is a wider age range in Charts VI and VII than there was in the first five, which tends to raise the correlation. For a single age level, however, our correlations are higher than those in any of the charts.

error. It is only in extreme cases of over and under weight that we fail to predict normal body-weight within reasonable limits of error, but, as we have stated previously, it is only in these obviously abnormal† cases that the method presented here would not be required. Large variations which are not due to skeletal size would be readily seen.

TABLE 4. PERCENTAGE OF CASES IN WHICH THE DISCREPANCIES BETWEEN ACTUAL AND PREDICTED WEIGHTS ARE OF THE SIZE INDICATED. FIFTY BOYS AND FIFTY GIRLS MEASURED AT AGES FOURTEEN TO EIGHTEEN BY THE HARVARD GROWTH STUDY

Age	N	Size of Discrepancies. Actual minus Predicted Weight (Kilos*)	Percentage (Girls)	Cumulative Percentages	Percentage (Boys)	Cumulative Percentages
14	50	0 –2.3	46	46	50	50
		2.4–4.6	26	72	24	74
		4.7–6.9	18	90 ($N=50$)	18	92
		7.0 or greater	10	100	8	100
15	50	0 –2.3	32	32	40	40
		2.4–4.6	34	66	28	68
		4.7–6.9	20	86 ($N=50$)	22	90
		7.0 or greater	14	100	10	100
16	50	0 –2.3	32	32	42	42
		2.4–4.6	26	58	28	70
		4.7–6.9	22	80 ($N=50$)	14	84
		7.0 or greater	20	100	16	100
17	50	0 –2.3	30	30	34	34
		2.4–4.6	34	64	26	60
		4.7–6.9	16	80 ($N=50$)	22	82
		7.0 or greater	20	100	18	100
18	25	0 –2.3	28	28	35	35
		2.4–4.6	24	52	26	61
		4.7–6.9	20	72 ($N=31$)	29	90
		7.0 or greater	28	100	10	100

* The groupings by kilos are made to correspond approximately to 0-to-5 pounds, 6-to-10 pounds, 11-to-15 pounds, and over 15 pounds.

The same subjects were used at all age levels. At age eighteen, twenty-five girls and nineteen boys had left school, and were threfore not measured.

As a final step in determining the superiority of our method over that of the height-weight tables, we have computed the correlation between weights as

TABLE 5. CORRELATION COEFFICIENTS BETWEEN ACTUAL AND PREDICTED WEIGHT. ALSO BETWEEN ACTUAL WEIGHT AND WEIGHT READ FROM THE BALDWIN-WOOD TABLES. MEASUREMENTS OF FIFTY BOYS AND FIFTY GIRLS AT AGES INDICATED. DATA FROM THE HARVARD GROWTH STUDY

Variables	r_{ap}	No. of Measurements	Sex	Age
Actual Weight and Predicted Weight	.911	215*	Boys	14–18
Actual Weight and Height-Weight Table Reading	.781	250	Boys	14–18
Actual Weight and Predicted Weight	.842	219	Girls	14–18
Actual Weight and Height-Weight Table Reading	.670	250	Girls	14–18

* Complete measurements were not available for all cases at all ages.

† And abnormality must now be considered with respect to height, width of iliac, and chest measurements.

obtained from the Baldwin-Wood Tables, actual measured weights, and our predicted weights. The results are reported in Table 5, an examination of which reveals that the correlation between actual measured weight and weight as predicted by formula is .911 for 215 boys' measurements at ages fourteen to eighteen, and .842 for girls', while the correlation between actual weight and

CHART VI

Predicted Weight (Kgs.) Boys

Actual Weight	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81
81																		1
78																		
75												1		1	2			
72												1	2	2	2			
69												6	1	1	3			
66										3	5	6	3	1	1			
63									1	7	7	6	7	1				
60									3	5	4	1	1					
57								5	8	3	7	5	1					
54							1	7	6	8	1	2						
51						2	3	4	1	2	3							
48					3	4	4	3	4									
45					3	3	3	1										
42			1	1	3	6	2	1										
39				2	5	2												
36	2	1	2	1														
33	2	2																
30																		

CHART VII

Predicted Weight (Kgs.) Girls

Actual Weight	30	33	36	39	42	45	48	51	54	57	60	63	66	69
69											1			
66											3			
63										2	3	1		
60									1	8	3	1		
57							2	3	9	11	6			
54						1	8	7	13	5	2			
51					1	5	14	4	5	5				
48					5	6	14	13	1	1				
45				1	5	8	6	2	1					
42				2	6	6	1							
39			2	1	4	3								
36			2	1	1									
33		1		1										
30		1												

weight from the Height-Weight Table is .781 for the boys and .670 for the girls. Now the difference between the use of the table and the use of the formula does not seem large until we look into the effectiveness of both techniques in forecasting what an individual should weigh.* A correlation of .911 indicates that we can forecast weight correctly by use of our formula with an effectiveness of 58.5 percent. By the same procedure we find that the forecasting efficiency of

* We do not use the terms "forecasting" or "predicting" body-weight, yet that is essentially what we are doing when we use a height-weight table. We say that a child of a certain height *should* weigh a certain number of pounds.

the Height-Weight Table is only 37.4 percent. Thus our formula is 21 percent more effective than the Height-Weight Table in forecasting the weight of boys aged 14 to 18. In the case of girls of the same age we find our formula to be 20 percent more effective than the Height-Weight Table.

The equation for the prediction of normal body weight which we have presented here is not too complex for ready acceptance as a substitute for the customary single measure of height. Its superiority over the use of the simple

TABLE 6. PREDICTED WEIGHT IN KILOS AND POUNDS OF SUBJECTS HAVING GIVEN MEASUREMENTS

Chest Depth	Chest Width	Standing Height	Iliac Width	Weight	
				Kilos	Pounds
14.8	22.8	162.5	25.0	40.8	90.0
15.0	23.0	163.0	25.2	42.0	92.6
15.2	23.2	163.5	25.4	43.4	95.7
15.4	23.4	164.0	25.6	44.5	98.1
15.6	23.6	164.5	25.8	45.8	101.0
15.8	23.8	165.0	26.0	46.9	103.4
16.0	24.0	165.5	26.2	48.1	106.1
16.2	24.2	166.0	26.4	49.4	108.9
16.4	24.4	166.5	26.6	50.7	111.8
16.6	24.6	167.0	26.8	51.9	114.4
16.8	24.8	167.5	27.0	53.1	117.1
17.0	25.0	168.0	27.2	54.3	119.7
17.2	25.2	168.5	27.4	55.6	122.6
17.4	25.4	169.0	27.6	56.8	125.2
17.6	25.6	169.5	27.8	58.0	127.9
17.8	25.8	170.0	28.0	59.2	130.5
18.0	26.0	170.5	28.2	60.4	133.2
18.2	26.2	171.0	28.4	61.7	136.0
18.4	26.4	171.5	28.6	62.9	138.7
18.6	26.6	172.0	28.8	64.2	141.6
18.8	26.8	172.5	29.0	65.3	144.0
19.0	27.0	173.0	29.2	66.6	146.8
19.2	27.2	173.5	29.4	67.8	149.5
19.4	27.4	174.0	29.6	69.1	152.4
19.6	27.6	174.5	29.8	70.3	155.0
19.8	27.8	175.0	30.0	71.3	157.2
20.0	28.0	175.5	30.2	72.7	160.3
20.2	28.2	176.0	30.4	73.9	162.9
20.4	28.4	176.5	30.6	75.2	165.8
20.6	28.6	177.0	30.8	76.4	168.5

measure of height would justify its use. The number of possible combinations of the four measurements used in our formula are so many in any large group of subjects that the construction of a table which could be readily substituted for the Height-Weight Table has not yet been accomplished. We have, however, made up a table using intervals of one-half centimeter for height and two-tenths centimeters for the other measurements, have performed the operations required and have drawn them up in a table to illustrate the procedure. In any institution where large groups of subjects are measured, it would be possible to collect various possible combinations for frequent use. It should be noted, also, that the operations are of a routine nature and are such that they can be carried out by any clerk who can multiply, or better still, can use a computing

machine. For the isolated case, as when a physician is examining a patient, the simple task of calculating normal weight by use of the formula would appear to be worth while, since the effort would result in a more accurate estimate and we advise its use. Even if our formula is not used in actual practice, however, it should be of some value in reminding people that width and depth must be considered in determining the extent to which an individual is above or below normal weight.

The results of our thinking and study of the problems of body-build in relation to body-weight lead us to question the validity of numerous studies, seeking to determine the relationships existing between extent of variation from normal weight (as determined by height-weight tables), personality development, performance on intelligence and school tests, socio-economic status, and diet, to name but a few factors which have been considered. We shall report the results of studies of the relationship between some of these variables and discrepancies from normal body-weight as determined by our formula in a second paper.

SUMMARY

1. Our study of the measurements of 533 sixteen-year-old boys indicates that it is impossible to classify such individuals into body types with any substantial degree of accuracy.

2. Any measure or index of body build which omits measures of width and depth of the body (and if used, fails to combine them properly) fails to give due weight to such measures, and is therefore inadequate for description of body build.

3. We have devised an equation for the prediction of normal body weight, which is 21 percent more efficient in the case of boys aged 14 to 18, and 20 percent more efficient in the case of girls of the same age than the Height-Weight Table in the determination of normal weight.

4. We suggest that our equation be substituted for use in the place of the usual height-weight tables in determination of the extent to which an individual is over or under weight in relation to measures of the width and depth of the chest, width of iliac, and height. The equation is easy to use, and the increase of labor required is justified by its additional accuracy.

5. Our equation is suitable for both sexes from years 14 to 18. For ages below these, it will be necessary to construct an equation or other equations of like nature.

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